MAE105
Quiz #4
(closed book, closed notes, no computer, no calculator)

Name:_____

Time: 3:35 to 3:55pm

Date: May 8, 2008

Consider the following ODE:

$$\frac{d^2\phi}{dx^2} + \lambda^2\phi = 0, \qquad -\pi < x < \pi\,. \tag{1}$$

(a) (0.5 Point) By direct substitution, show that $\phi = a\cos(\lambda x)$ is a solution of (1).

(b) (0.5 Point) By direct substitution, show that $\phi = b\sin(\lambda x)$ is a solution of (1).

(c) (0.5 Point) By direct substitution, show that $\phi = a\cos(\lambda x) + b\sin(\lambda x)$ is a solution of (1).

Consider for (1) the following continuity conditions:

$$\phi(-\pi) = \phi(\pi), \qquad \frac{d\phi}{dx}(-\pi) = \frac{d\phi}{dx}(\pi)\,. \tag{2}$$

(d) (1.5 Point) Use these continuity conditions to find explicit expressions for the eigenvalues, λ_n, and the corresponding eigenfunctions, $\phi_n(x)$.

Note: You must use BOTH continuity conditions and show whether or not both a and b in the expression for $\phi(x)$, given in (c) above, are nonzero.

(e) (2 Points) Consider the function

$$f(x) = \pi - x\,, \qquad -\pi < x < \pi\,. \tag{3}$$

Find its Fourier representation in terms of the eigenfunctions in Part (d), and calculate all the unknown constants explicitly.

[Note: Use integration by parts.]

Note: To receive full credit, *all steps must be neatly shown, following the requested procedure.* Writing down the final results will receive no credit.

Good luck.