

MAE105 Quiz #1
(closed book, closed notes, no computer, no calculator)

Name: _____

Time: 3:35 to 3:55pm

Date: April 10, 2008

(a) (1 Point) Use the separation of variables, $u(x, t) = \phi(x)G(t)$, to find two ODE's, one defining the time variation and the other the spatial variation of the general solution of the following PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < \pi,$$

in such a way that the solution for $G(t)$ would DECAY in time.

(b) (1 Point) Solve the two ODE's to find the general expressions for $G(t)$ and $\phi(x)$.

Consider the following boundary conditions:

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad \frac{\partial u}{\partial x}(\pi, t) = 0.$$

(c) (0.5 Point) Apply these boundary conditions to find the corresponding boundary conditions for $\phi(x)$.

(d) (1 Point) Find the eigenvalues and eigenfunctions corresponding to $\phi(x)$. Is zero also an eigenvalue in the present case? If so, then what is the corresponding eigenfunction?

(e) (0.5 Point) Write down the general infinite series solution for $u(x, t)$.

(f) (1 Point) Obtain the final solution that satisfies the following initial condition and write down the final solution:

$$u(x, 0) = 3 + 5 \cos(7x).$$

Note 1: To receive full credit, *all steps must be neatly shown, following the requested procedure*. Writing down the final results will receive no credit.

Note 2: Once you have finished, please **stay in your seat** and the Instructor will collect your paper at the end of the quiz time.