

MAE105
Homework #4, Due Tuesday April 22, 2008

Name: _____

Date: April 15, 2008

Problem 1

(6 Points) Solve the Laplace's equation inside a quarter circle,

$$\nabla^2 u = 0, \quad 0 < r < a, \quad 0 < \theta < \pi/2, \quad (1.1)$$

with boundary conditions

$$u(r, 0) = 0, \quad u(r, \pi/2) = 0, \quad u(a, \theta) = (\theta - \pi/2) \sin \theta. \quad (1.2)$$

Problem 2

Consider the following PDE:

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0. \quad 0 < x < \pi, \quad t > 0. \quad (2.1)$$

(a) (1 Point) Use the separation of variables to find two ODE's, one to define the dependence of u on x , and the other for the dependence of u on t . Show all the details.

(b) (1 Point) Integrate the ODE that gives the dependence of u on t , for *all possible cases*. Show all the details.

(c) (1 Point) For *all three cases* in (a), integrate the second ODE in (a) and obtain the spatial dependence of u . Show all the details.

(d) (0.5 Point) Write down the general solution of the original PDE (3.1).

(e) (1 Point) Consider the following boundary conditions for PDE (3.1):

$$\frac{\partial u}{\partial x}(0, t) = 0, \quad u(\pi, t) = 0. \quad (2.2)$$

Find the infinite series solution which *does not grow in time*.

(f) (1.5 Points) For the initial condition

$$u(x, 0) = x \cos(x), \quad (2.3)$$

find the complete solution.

Show all steps and the details.