

April 10, 2008

**MAE 105 Homework #3**  
**Due: Tuesday, 04/15/08**

**PROBLEM 1:**

Consider the heat equation,

$$\frac{\partial u}{\partial t} - 2 \frac{\partial^2 u}{\partial x^2} = 0 \quad t > 0, \quad 0 < x < 2\pi.$$

a. (1 Point): Use the separation of variables,  $u(x, t) = G(t) \phi(x)$ , to find the necessary ODEs, one for  $G(t)$  and another for  $\phi(x)$ . Choose your ODEs such that the solution for  $\phi(x)$  is periodic.

b. (2 Points): Find  $G(t)$  and  $\phi(x)$ .

c. (1 Point): Consider the boundary conditions,

$$u(0, t) = u(2\pi, t), \quad \frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(2\pi, t),$$

Write the corresponding boundary conditions for  $\phi(x)$ .

d. (2 Points) Find all the eigenvalues and the corresponding eigenfunctions, using the boundary conditions.

e. (2 Points) Find the complete solution for  $u(x, t)$  when the initial condition is given by

$$u(x, 0) = 2 + 4 \sin 3x - 2 \cos 9x.$$