

April 8, 2008

MAE 105 Homework #2

Due: Tuesday, 4/15/08

PROBLEM 1 (2 Points):

Use the identity, $e^{\pm i\theta} = \cos \theta \pm i \sin \theta$ to express $\sin \alpha \sin \beta$, $\sin \alpha \cos \beta$, $\cos \alpha \cos \beta$, in terms of $\sin(\alpha \pm \beta)$, and $\cos(\alpha \pm \beta)$.

PROBLEM 2 (1.5 Points):

Use the results from PROBLEM 1, to explicitly integrate the following definite integral:

$$I_1 = \int_0^x \cos(\alpha x') \cos(\beta x') dx', \quad I_2 = \int_0^x \sin(\alpha x') \sin(\beta x') dx',$$

$$I_3 = \int_0^x \cos(\alpha x') \sin(\beta x') dx'.$$

PROBLEM 3 (6 Points):

Consider the following ODE:

$$\frac{d^2 \phi}{dx^2} + \alpha \phi = 0, \quad 0 < x < \pi. \tag{B-1}$$

- (a) Find the general solution when $\alpha = 4$.
- (b) Find the general solution when $\alpha = -4$.
- (c) Find the general solution when $\alpha = 0$.

[Note 1: Each of your general solution must include two integration constants.]

For each of the following boundary conditions, viewed as separate problem, find the complete solution of (B-1) for all cases, (a), (b), and (c) above:

$$(A): \quad \phi(0) = \phi(\pi) = 0. \tag{B-2}$$

$$(B): \quad \frac{d\phi}{dx}(x=0) = 0, \tag{B-3}$$

$$\phi(\pi) = 0.$$

$$(C): \quad \phi(0) = 0, \tag{B-4}$$

$$\frac{d\phi}{dx}(x=\pi) = 0.$$

Note 2: To receive full credit, *all steps must be neatly shown*. Writing down the final results will receive no credit.

Note 3: Homeworks must be turned in at the start of due-date class. Late homeworks will be graded but *will receive zero credit*.