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The Effect of Stable Stratification on Turbulence Anisotropy in Uniformly Sheared Flow

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Abstract—Direct numerical simulation of uniform shear flow is used to study the anisotropy of fluctuating motion in a stably stratified medium with uniform mean shear. Turbulence is found to be three dimensional over a wide range of gradient Richardson numbers in the two flows investigated here: vertical mean shear ($\frac{d\bar{U}}{dz}$) and horizontal mean shear ($\frac{d\bar{U}}{dy}$). The role of the turbulent Froude number in establishing the regime of stratified turbulence observed here is described. The fluctuating velocity gradients are examined. The vertical of streamwise velocity is found to dominate the other components of turbulent dissipation in both horizontal and vertical shear flows.

Keywords—Stratified turbulence, Geophysical flows, Direct simulation, Ocean microstructure.

1. INTRODUCTION

Stably stratified shear flow with horizontal mean shear, $\frac{d\bar{U}}{dy} = S_h$, was compared with vertical mean shear, $\frac{d\bar{U}}{dz} = S_v$ using DNS (direct numerical simulation) in our previous studies, [1,2]. The turbulent kinetic energy was found to be much larger when the mean shear is horizontal essentially because the turbulent production in such a case does not directly involve the gravity-suppressed vertical velocity and the associated vertical buoyancy flux is larger because turbulence remains three dimensional. However, gravity was found to affect the overall dynamics, for example, when $\mathrm{Ri}_g = N^2/S_h^2$ was larger that a critical value of $\mathrm{Ri}_g \simeq 1.5$, the turbulence was found to decay. The coupling between fluctuations in horizontal velocity, vertical velocity, and density is key to understanding the observed behavior and motivates the present study of turbulence anisotropy. Presumably, the coupling leads to qualitative differences in the vertical mixing due to different types of fluctuations supported in the stably-stratified ocean such as internal waves [3], buoyancy-affected but three-dimensional turbulence, the potential vorticity mode [4,5], and fossil turbulence [6].

In the case of vertical shear flow, it is well known that, with increasing Richardson number, the turbulent energy and associated vertical mass transport is suppressed. However, the effect of stratification on the anisotropy of the fluctuating velocity is not clear. Ocean spectra, for example [7], often show internal wave activity at low wave numbers followed by three-dimensional

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turbulence. In the case of decaying grid turbulence, the laboratory experiments of Pearson and Linden [8], Lienhard and van Atta [9], Fincham *et al.* [10], and DNS of Metais and Herring [11] indicate that, although the vertical component is reduced with respect to the other components, it is only after a very long time that the state is close to the two-dimensional (more accurately two-component) limit of horizontal turbulence. In the case of uniform vertical shear, the laboratory studies of Rohr *et al.* [12] and DNS of Gerz *et al.* [13], Holt *et al.* [14], Jacobitz and Sarkar [15] indicate that the two-component limit is not reached. The results of Itsweire *et al.* [16] find significant deviation of small-scale isotropy due to stratification. Laboratory investigations of wakes by Chomaz *et al.* [17], Spedding *et al.* [18], and jets by Voropayev *et al.* [19] show the formation of predominantly horizontal eddying motion. The studies of Fincham *et al.* [10] and Spedding *et al.* [18] suggest that the vertical variability of the collapsed horizontal structures lead to dominant velocity gradients in the vertical direction.

2. THE GOVERNING EQUATIONS

The density ρ, the velocity u_i, and the pressure p, denote fluctuations with respect to the mean density $\bar{\rho}$, the mean velocity $\bar{U}_i$, and the mean pressure $\bar{p}$. The uniform mean density gradient, $\frac{\partial \bar{\rho}}{\partial x_3} = S_\rho$, imposes a stable stratification which is hydrostatically balanced by a corresponding mean pressure gradient. Uniform mean shear, $\frac{\partial \bar{U}_1}{\partial x_3} = S_v$ or $\frac{\partial \bar{U}_1}{\partial x_2} = S_h$, provides the forcing for turbulence. The effect of rotation is neglected since, for the scales considered here, the Rossby number $\mathrm{Ro} \gg O(1)$.

After the customary Boussinesq assumption, the equations governing the evolution of the fluctuating variables are as follows,

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{Du_1}{Dt} + u_j \frac{\partial u_1}{\partial x_j} = -S_v u_3 - S_h u_2 - \frac{1}{\rho_0}\frac{\partial p}{\partial x_1} + \nu \frac{\partial^2 u_1}{\partial x_j \partial x_j}, \tag{2}$$

$$\frac{Du_2}{Dt} + u_j \frac{\partial u_2}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_2} + \nu \frac{\partial^2 u_2}{\partial x_j \partial x_j}, \tag{3}$$

$$\frac{Du_3}{Dt} + u_j \frac{\partial u_3}{\partial x_j} = -g\frac{\rho}{\rho_0} - \frac{1}{\rho_0}\frac{\partial p}{\partial x_3} + \nu \frac{\partial^2 u_3}{\partial x_j \partial x_j}, \tag{4}$$

$$\frac{D\rho}{Dt} + u_j \frac{\partial \rho}{\partial x_j} = -S_\rho u_3 + \alpha \frac{\partial^2 \rho}{\partial x_j \partial x_j}, \tag{5}$$

where ν and α are molecular transport coefficients. Examination of equations (1)–(5) shows a highly anisotropic structure. Mean shear directly forces only the streamwise velocity while the other velocity components are forced by the fluctuating pressure gradient. Gravity and density fluctuations directly influence only the vertical component u_3. The mean density gradient appears only in the density fluctuation equation and not the mean momentum equation.

In the absence of stratification, all three components of the fluctuating velocity in a shear flow are experimentally observed to have comparable magnitudes with the following ordering: streamwise > spanwise > cross-stream. In the case of uniform vertical shear, equilibrium values of the energy partition into the three components that are experimentally measured when $\mathrm{Ri}_g = 0$ are

$$\overline{u_1}^2/2K = 0.53, \qquad \overline{u_2}^2/2K = 0.27, \qquad \overline{u_3}^2/2K = 0.20. \tag{6}$$

Despite the fact, that mean shear forces only a single velocity component, the other two velocities are induced to have comparable magnitudes by the fluctuating pressure gradient since the fluctuating pressure is linked to *all* velocity components by the following Poisson equation:

$$\frac{\nabla^2 p}{\rho_0} = -2S_v \frac{\partial u_3}{\partial x_1} - \frac{\partial u_j}{\partial x_i}\frac{\partial u_i}{\partial x_j} + \frac{g}{\rho_0}\frac{\partial \rho}{\partial x_3}. \tag{7}$$

The buoyancy term in the momentum equation introduces an additional nondimensional parameter into the problem. If the density fluctuation is estimated by $\rho \simeq (\frac{\partial \bar{\rho}}{\partial x_3})l_3$ where l_3 is the appropriate vertical length scale, then the turbulent Froude number,

$$\mathrm{Fr}_t = u'/Nl_3, \tag{8}$$

appears naturally as the relevant parameter because the ratio of the two terms that cause vertical motion, the pressure gradient and buoyant acceleration, is $O(\mathrm{Fr}_t^2)$. Buoyancy effects are important when $\mathrm{Fr}_t \leq O(1)$. Here, $N = \sqrt{-gS_\rho/\rho_0}$ is the Brunt-Vaisala frequency while the superscript $'$ denotes the characteristic scale for a fluctuation, for example, u' is a typical magnitude of velocity perturbation. In the case of sheared turbulence, the gradient Richardson number, $\mathrm{Ri}_g = N^2/S^2$, is a convenient choice for parametric studies because it quantifies the ratio of mean buoyant forcing to mean shear forcing. Setting aside the issue of convenience, it should be emphasized that it is the Froude number, Fr_t, which fundamentally determines the dynamical coupling between stratification and fluctuating motion.

3. DNS RESULTS ON THE VELOCITY ANISOTROPY

It is clear, that the vertical component of turbulence is suppressed with increasing stable stratification since turbulence has to do work against gravity. This gravity-induced reduction of vertical motion eventually leads to global suppression of turbulence in stratified shear flow. For example, in the DNS cases to be considered here, the turbulent kinetic energy K decays when $\mathrm{Ri}_{g,cr} \simeq 0.18$ is exceeded in vertical shear flow and $\mathrm{Ri}_{g,cr} \simeq 1.5$ is exceeded in horizontal shear flow. Reduction of the vertical component is often thought to imply that turbulence tends to the two-component limit with the vertical velocity fluctuation much smaller than the horizontal velocity fluctuations. Figure 1a shows the evolution of vertical energy partition $\overline{u}_3^2/2K$ in vertical shear flow. In the case with $\mathrm{Ri}_g = 0$, after an initial transient, the experimentally observed value shown in equation (6) is asymptotically approached. The vertical energy partition decreases somewhat with increasing Richardson number. However, the two-component limit is certainly not reached. Figure 1b shows that, in horizontal shear flow too, the turbulence does not reach a two-component state.

4. THEORETICAL SCALINGS APPLICABLE TO THE VELOCITY COMPONENTS

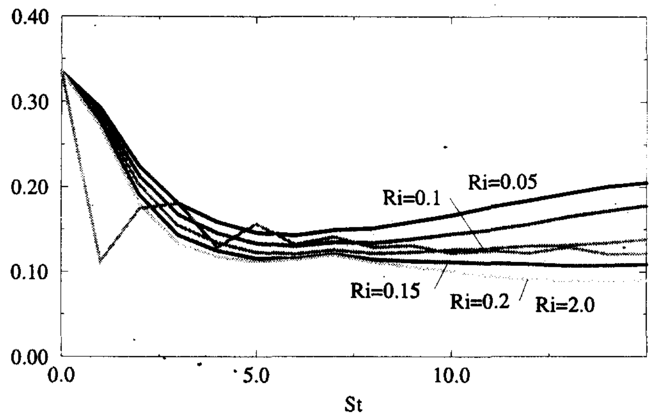
In order to understand our results, we reconsider the governing equations to derive simple order-of-magnitude estimates applicable when buoyancy substantially effects the state of fluctuating motion. If the other term in equation (4) that balances buoyancy is the unsteady term, then

$$\frac{u_3'}{\tau_3} = O\left(g\frac{\rho'}{\rho_0}\right), \tag{9}$$

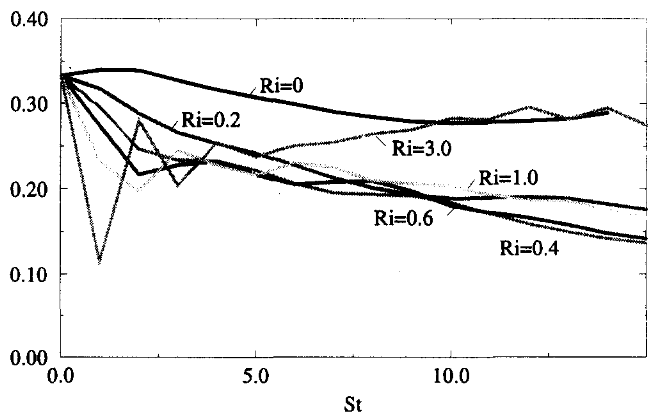
where τ_3 is the characteristic time scale. Similarly, if the unsteady term balances the forcing by mean density gradient in equation (5) then,

$$\frac{\rho'}{\tau_3} = O\left(u_3' S_\rho\right). \tag{10}$$

Multiplying equations (9) and (10) leads to the following relation for the time scale $\tau_3 = 1/N$. The magnitude of u_3 is set by the fluctuating pressure gradient in the vertical momentum balance and estimating the pressure by $p = O(\rho_0 u'^2)$, leads to $u_3'/u' = O(\mathrm{Fr}_t)$. Recall that the Froude number Fr_t is defined by equation (8). The nonlinear terms do not invalidate these scalings as long as they are of the same or lesser order as the unsteady term. Thus, the vertical velocity has a characteristic time scale N, imposed by stratification.



(a)



(b)

Figure 1. Evolution of the vertical energy partition $\overline{u_3^2}/2K$ as a function of the gradient Richardson number Ri in (a) vertical shear flow and (b) horizontal shear flow.

The streamwise velocity fluctuation on the other hand, has a characteristic time scale $1/S$, imposed by the mean shear. It should be noted that the fluctuating pressure gradient couples the vertical and horizontal motions by imposing the time scale $1/S$, on u_3, and similarly, $1/N$ on u_1 and u_2. It can be anticipated, that with increasing values of $\mathrm{Ri}_g = N^2/S^2$, the vertical shear-induced correlation between u_1 and u_3 as well as the horizontal shear-induced correlation between u_1 and u_2 is hampered because of the increasing disparity between the characteristic time scales, $1/S$ and $1/N$. The reduction in shear stress, $\overline{u_1 u_3}$ and $\overline{u_1 u_2}$, and associated production of turbulent kinetic energy K leads to eventual decay of velocity fluctuations for sufficiently large stratification. Indeed, the direct cause of turbulence decay in a stratified medium is the reduced production and not the buoyancy flux. In summary, the regime of stratified sheared turbulence which occurs when $\mathrm{Fr}_t \leq O(1)$ and *all* terms (except the viscous term) are of the same order in the vertical momentum equation, is characterized by the following scalings for the fluctuating variables:

$$\frac{u_3'}{u_1'} = O(\mathrm{Fr}_t), \qquad \frac{\rho'}{\rho_0} = \frac{u'^2}{g l_3}, \qquad N\tau_3 = O(1). \tag{11}$$

It is emphasized that, although the motion described by equation (11) has a characteristic time scale $O(1/N)$ for the vertical velocity, this mode does *not* necessarily represent a propagating linear internal wave.

In the preceding arguments, the assumption that the unsteady term balances the buoyancy term in equation (4) leads to the constraint that $\tau_3 = O(1/N)$. A plausible alternative is to fix the time scale τ in all equations to be the horizontal advection time scale l_1/u_1'. Furthermore, if $\mathrm{Fr}_t \ll 1$, the dominant balance in the u_3 equation *must* be between the fluctuating pressure gradient and the buoyancy term with the unsteady term being of higher order. As shown by Riley and Lelong [20], such a balance leads to the potential vorticity mode characterized by

$$\frac{l_3}{l_1} = O(\alpha), \qquad \frac{u_3'}{u_1'} = O\left(\alpha\,\mathrm{Fr}_t^2\right), \qquad \frac{\rho'}{\rho_0} = O\left(\frac{u'^2}{gl_3}\right). \tag{12}$$

Here, α is the aspect ratio introduced by the system geometry. The current discussion is limited to geophysical length scales of $l < 50\,\mathrm{m}$ with $\alpha = O(1)$. According to both equations (11) and (12), the fluctuating velocity approaches the two-component limit $u_3'/u_1' \to 0$, *assuming* $\mathrm{Fr}_t \to 0$ with increasing stratification.

5. THE TURBULENT FROUDE NUMBER

Since the relative magnitude of vertical and horizontal components is controlled by the turbulent Froude number, it is of interest to obtain the evolution of Froude number from our DNS. It is found that Fr_t asymptotically reaches a constant in the simulations. The dependence of the asymptotic value of the vertical Froude number $\mathrm{Fr}_w = w'/NL_e$ on the gradient Richardson number is given in Figure 2. Here w' is the r.m.s. vertical velocity fluctuation while $L_e = \rho_{\mathrm{rms}}/(\frac{d\bar{\rho}}{dx_3})$ is the Ellison scale. The most striking aspect of Figure 2 is that, after a rapid initial decrease, the Froude number becomes relatively *independent* of Ri_g with $\mathrm{Fr}_w \simeq 0.6$ in vertical shear flow and $\mathrm{Fr}_w \simeq 0.75$ in horizontal shear flow. There appears to be no tendency for $\mathrm{Fr} \to 0$ explaining our DNS result that the fluctuating motion does not approach the two-component limit even for large values of Ri_g corresponding to strongly-decaying turbulence.

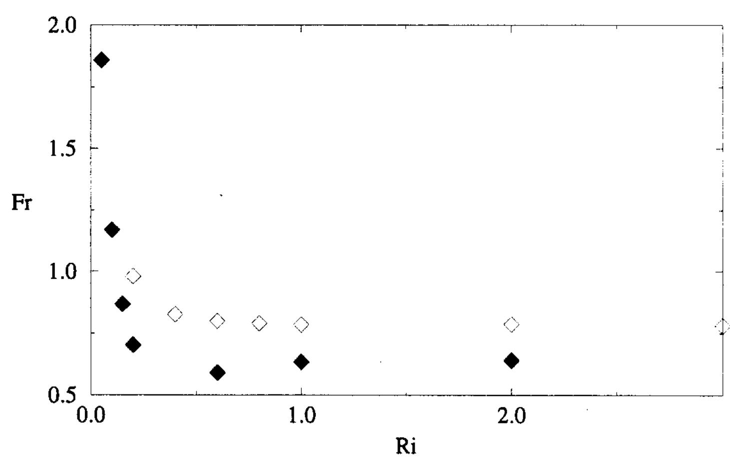


Figure 2. The variation of Fr_t as a function of gradient Richardson number in vertical shear flow (filled symbols) and horizontal shear flow (unfilled symbols).

Why does the turbulent Froude number not approach zero with increasing values of N? The unsteady term and the shear-forcing term must be of the same order in the streamwise momentum balance, equation (2), giving,

$$\frac{u_3}{u_1} = O\left(\frac{1}{S\tau_1}\right). \tag{13}$$

The l.h.s. of equation (13) was derived to be $O(\mathrm{Fr}_t)$ and, since u_1 is forced by the mean shear, the r.h.s. is $O(1)$. Thus,

$$\mathrm{Fr}_t = O(1). \tag{14}$$

6. ANISOTROPY OF THE VELOCITY GRADIENT

Stratification imposes anisotropy on the velocity gradients, and consequently, the turbulent dissipation. Table 1 shows values of the nine components, $\varepsilon_{\alpha\beta}$, defined by

$$\varepsilon_{\alpha\beta} = \frac{\nu \overline{\frac{\partial u_\alpha}{\partial x_\beta} \frac{\partial u_\alpha}{\partial x_\beta}}}{\varepsilon}. \tag{15}$$

Note that there is no summation over any Greek index and the turbulent dissipation rate ε is used for normalization. For reference, a corresponding table in the case of isotropic turbulence would show the value of 0.0667 for all straining components ($\alpha = \beta$) and a value of 0.1333 for all shearing components. Table 1 shows the relative contributions for both zero and a large value of Ri_g.

Table 1. The component $\varepsilon_{\alpha\beta}/\varepsilon$ at $St = 11$ in (a) horizontally sheared flow, and (b) vertically sheared flow.

(a)

$\mathrm{Ri}_g = 0.00$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
$\beta = 1$	0.045	0.049	0.068
$\beta = 2$	0.218	0.060	0.190
$\beta = 3$	0.186	0.116	0.069

$\mathrm{Ri}_g = 3.00$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
$\beta = 1$	0.005	0.003	0.009
$\beta = 2$	0.099	0.058	0.271
$\beta = 3$	0.414	0.078	0.062

(b)

$\mathrm{Ri}_g = 0.00$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
$\beta = 1$	0.045	0.065	0.045
$\beta = 2$	0.193	0.069	0.113
$\beta = 3$	0.219	0.192	0.060

$\mathrm{Ri}_g = 2.00$	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$
$\beta = 1$	0.009	0.007	0.001
$\beta = 2$	0.087	0.021	0.023
$\beta = 3$	0.470	0.361	0.020

In the unstratified case with $\mathrm{Ri}_g = 0$, the largest gradient corresponds to the shearing direction, the 1–2 component in horizontal shear flow and the 1–3 component in vertical shear flow. However, for large Ri_g, the 1–3 component, which is the vertical gradient of the streamwise fluctuation, dominates all other components. In the flow with vertical mean shear, the case with high Richardson number has decaying turbulence and one might suspect that the dominance of the 1–3 fluctuating gradient is due to low Reynolds number which allows the anisotropy of the mean distortion to permeate into the small scales. However, the observed dominance of the 1–3 fluctuation at high Ri_g in the case of horizontal mean shear, $\frac{\partial \bar{U}_1}{\partial x_2}$, cannot be a low Reynolds number artifact. Evidently, it is the stable stratification that causes the dominance of the fluctuating vertical shear irrespective of the direction of mean shear forcing. Recent PIV measurements in stratified wakes and grid turbulence have been used to infer collapse of the motion into horizontal layers accompanied by large vertical gradients due to decorrelation of these horizontal layers in the vertical direction. Our observation of the dominance of the fluctuating vertical shear in uniformly sheared flow is consistent with these experimental observations.

Stable stratification induces other changes in the small-scale anisotropy. In the case of vertical mean shear, the 2–3 component increases strongly while the 3–2 component decreases strongly. In the case of horizontal mean shear, the 3–2 component increases while the 2–3 component decreases. Such behavior suggests that, in both cases, the relative contribution of the streamwise vorticity component increases. Similarly, it can be deduced that the x_2 vorticity component also increases while the x_3 vorticity component decreases. Diamessis and Nomura [21] have observed a predominance of horizontal fluctuating vorticity in their DNS of stratified flow forced by vertical mean shear. The current results are consistent with the observations of Diamessis and Nomura [21].

8. CONCLUSIONS

In the case of uniformly sheared flow with either horizontal $(\frac{d\bar{U}}{dy})$ or vertical $(\frac{d\bar{U}}{dz})$ shear, horizontal and vertical velocity fluctuations are found to remain coupled over the range of Richardson numbers, $0 < \mathrm{Ri}_g < 3$, studied here. As a consequence of the coupling, first, horizontal mean shear induces vertical mixing, second, the vertical mixing is larger when the mean shear is horizontal instead of vertical since the turbulent production in that case is not directly inhibited by gravity, and third, the suppression of vertical fluctuations eventually leads to the overall decay of all velocity fluctuations for sufficiently large values of Ri_g. The anisotropy of fluctuating gradients is also found to be affected by stratification. At high Ri_g, in both horizontal and vertically sheared flows, the component $\frac{\partial u_1}{\partial x_3}$ dominates, while the streamwise gradients $\frac{\partial u_i}{\partial x_1}$ becomes small. It appears that, in response to stable stratification, fluctuating vertical shear dominates other components of the dissipation suggesting the appearance of decorrelated horizontal layers of motion. Furthermore, there is a collapse of the vorticity towards the horizontal plane.

The turbulent Froude number, Fr_t, is found to approach a $O(1)$ constant at high values of Ri. Thus, the limit of two-component turbulence which requires asymptotically small Froude number is *not* approached in the case of turbulence forced by uniform shear. There are other counterexamples such as the far-wake and horizontal jet where Fr_t progressively decreases. In such situations, the two-component limit may be approached. In geophysical flows with a variety of mechanisms available for forcing fluctuating motion, both three-dimensional and two-dimensional turbulence are potentially realizable in a stratified medium with the former responsible for most of the energetic vertical mixing.

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