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Optimal periodic disturbance reduction for active noise cancelation

C.E. Kinney[a], R.A. de Callafon[a,*], E. Dunens[b], R. Bargerhuff[b], C.E. Bash[b]

[a]Department of Mechanical and Aerospace Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0411, USA
[b]Hewlett-Packard Company, 20555 State Highway 249, Houston, TX 77070, USA

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Abstract

The design of an optimal internal model-based (IMB) controller by extending standard discrete time optimal control theory for IMB controllers is described. The optimal observer and state feedback gains of the IMB controller are given via the solution of discrete time algebraic Riccati equations. The design method is applied to an acoustic system that is subjected to disturbances from a server fan. Periodic disturbances from the server fan appear as harmonics of the fundamental frequency of the fan. Parametric models for the plant and non-periodic part of the disturbance are identified from experimental data. An internal model is designed in discrete time and the internal model principle is used to design a feedback controller that rejects periodic disturbances in the acoustic system. The controller is implemented in real-time and successfully attenuates the first four harmonics of the fan noise.

1. Introduction

1.1. Active noise control for cooling fans

Density in computer server design is a key criterion for customers. It permits greater use of valuable data center space, allowing more servers to be installed at one site to satisfy the demand for more computing power. The power requirements of many of the various features within the server have increased as the server enclosure's design envelope is reduced, see Ref. [1] for more information regarding cooling requirements and cooling technology for computers. As server components dissipate more heat, often their silicon cannot withstand higher junction temperatures and more cooling air is needed. Since the server packaging densities are higher, the resistance to additional airflow is also higher. In fact this airflow resistance, or static pressure, increases approximately as the square of the airflow required. While many improvements have been made in forced air-cooling systems, typically the rotational speed of the cooling fan blades is increased to help meet the

*Corresponding author. Tel.: +1 858 5343166; fax: +1 858 8223107.
E-mail address: callafon@ucsd.edu (R.A. de Callafon).

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need for more flow at higher static pressures. Many high performance fans currently used in the smallest servers are designed to rotate at 15,000 rev/min.

Clearly, a method of reducing the noise produced by the forced air-cooling systems that does not require a large amount of space nor have a significant cost impact is desired. Passively canceling noise usually entails mounting various materials onto or around the noisy area. These materials are often bulky, heavy, and do not dissipate heat easily. Therefore, active noise control [2–6] is an attractive option for this application, where there are weight and space constraints as well as cooling requirements. Additionally, the main source of noise is tonal due to the blade pass frequency (BPF) of the cooling fan and active noise control has been shown to work well in canceling these types of components [2,5].

1.2. Internal model-based (IMB) and repetitive control

Repetitive control [7–10] has been shown to effectively eliminate periodic disturbances. Typical repetitive control schemes include an internal model, often called a memory loop [11] in the repetitive control literature, is placed in the feedback loop in order to cancel the repetitive disturbance. Typically, two filters are used to modify the memory loop and guarantee stability of the closed loop system. One filter is used to create a stable internal model, and one filter is used to eliminate high-frequency components. This method results in a high order internal model, and the non-periodic effects are often left out of the analysis resulting in a controller that can over amplify these components.

Repetitive controllers can be viewed as an extension of the internal model principle [12]. The connection between the internal model principle and repetitive controllers has been discussed in Ref. [13]. Designing feedback controllers based upon the internal model principle has produced good results in the area of ANC, and the authors in Ref. [2] reported a 30–40 dB reduction in an acoustic duct. The internal model principle and convex optimization was used to find a controller that rejects periodic disturbances in Ref. [14]. The method contained therein requires that the plant has no zeros in common with the poles of the disturbance and optimizes the closed loop system over the zeros of the controller. In Ref. [15], a feedback controller was designed and implemented that successfully eliminated the first four harmonic frequencies of the disturbance. The repetitive control literature has also reported good results in the rejection of periodic disturbances. For example, in Ref. [16] a 25 dB reduction over a 25 Hz frequency range was demonstrated with a repetitive controller, and in Ref. [17] a successful implementation of a repetitive control algorithm on a non-minimum phase plant is described.

1.3. Contributions

The main contribution of this paper is to shown that the design of an IMB controller for the reduction of periodic disturbances in the presence of random disturbances can be solved by extending the standard linear quadratic Gaussian (LQG) control problem. As such, the control design reduces to finding the optimal observer and state feedback gains for the IMB controller in Ref. [18] in the presence of random noise. In this paper we show how the variance of the output, a key performance measure in systems with random and periodic disturbances, can be reduced with an LQG style of controller. In addition, the theoretical concepts are illustrated in an application of active noise control that includes identifying models for the control design, designing the internal model, and designing the controller.

Although this method presents an effective way of eliminating periodic disturbances in the presence of random noise, it does not allow for the case where the disturbance is time varying or unknown. That is, the type of controller designed here is non-scheduled and therefore stability will not be an issue as long as a suitable model of the plant is obtained but performance will suffer if the disturbance frequency is time varying or unknown.

2. Acoustic system

The acoustic system shown in Fig. 1 is used to demonstrate the effectiveness of the proposed control design. The system consists of an HP DL380 G3 rack server cooling fan mounted inside a acoustic duct that is lined

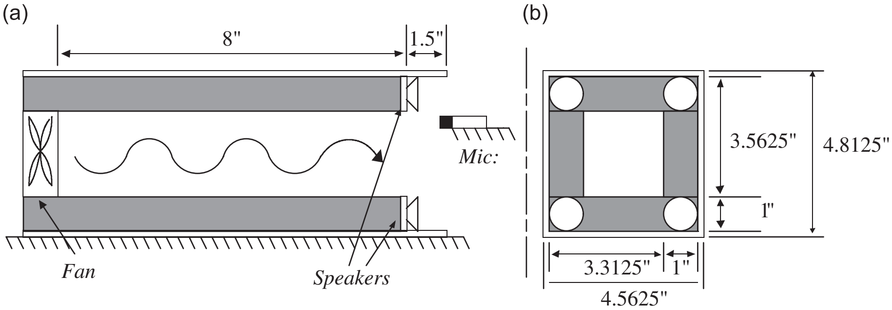

Fig. 1. Acoustic system used for test purposes of active noise control. The pick-up microphone is used for feedback control of the speakers mounted on the end of the acoustic duct: (a) side view and (b) end view.

with polyurethane acoustical foam, four speakers (1.1″ diameter) connected in parallel to act as one large speaker, and a pick-up microphone (a hands-free, tie-clip omnidirectional electret microphone) located at the end of the duct. The microphone is used as the feedback signal that is sent to a digital controller that is comprised of a Pentium based PC with a 12 bit signed analog to digital/digital to analog card.

The acoustic system is defined as the transfer function between the speakers and pick-up microphone. Typically, this system exhibits multiple resonances, several steps time delay, and is not stable for large feedback gain. The close proximity of the pick-up microphone to the speakers necessitates accurate modeling of the system's dynamics for stability issues; system identification will be used to this end.

Acoustic disturbances, composed of two types of signals: periodic and non-periodic (random), are created by the server fan. The periodic part of the disturbance signal is composed of multiple harmonics of the fundamental frequency that appear due to the blade passage frequency. The non-periodic part of the disturbance signal is broadband noise created by the turbulent air flow and vibrations moving down the duct. The effect of the non-periodic disturbances will be suppressed by designing an LQG controller with an appropriate cost function. The periodic disturbances will be rejected by using the internal model principle. Making the distinction between these two types of disturbances and designing a controller to cancel the periodic components in the presence of the non-periodic components is the focus of the techniques developed in this paper.

3. Internal model synthesis with discrete time harmonic oscillators

One method to design an internal model that can model the periodic components present in the fan noise is to use a series connection of lightly damped harmonic oscillators. In discrete time, the behavior of an oscillator can be described with the following equation:

$$x(k+2) + bx(k+1) + cx(k) = u(k). \tag{1}$$

The roots of the homogeneous part of the difference equation are for $4c < b^2$:

$$r_{1,2} = -\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}, \tag{2a}$$

while for $4c > b^2$

$$r_{1,2} = -\frac{b}{2} \pm j\sqrt{c - \frac{b^2}{4}} = |r|\mathrm{e}^{\pm \mathrm{j}\phi}, \tag{2b}$$

where $|r|$ is the radius of the discrete time poles and ϕ the angle of the poles in the complex z-plane. The radius of the roots $|r|$ determines how fast the solution will grow or decay, and the angle of the roots is actually the damped natural frequency of the oscillator scaled by the sampling rate. By using Euler's identity and basic properties of linear difference equations, the general solution can be expressed as

$$x(k) = |r|^k(A \cos \phi k + B \sin \phi k). \tag{3}$$

By using this information and knowledge about the general form of a continuous time second-order system ($s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$), a harmonic oscillator with a specific damping ratio ζ and an undamped natural frequency ω_n can be designed.

Given a desired natural frequency ω_n (rad/s), dampening ζ, and sampling rate f_s (Hz), the coefficient c is given by

$$c = \exp\left(-2\zeta \frac{\omega_n}{f_s}\right). \tag{4}$$

The desired angle ϕ is given by

$$\phi = \omega_n \sqrt{1-\zeta^2}\frac{1}{f_s}. \tag{5}$$

And lastly, b is given by

$$b = \pm\sqrt{\frac{4c}{(\tan \phi)^2 + 1}} = -2\sqrt{c}\cos(\phi), \tag{6}$$

where b is positive for angles between 90° and 180°, is a negative number for angles between 0° and 90°, and is zero at 90°.

Connecting several lightly damped oscillators together in series or parallel gives an internal model of the periodic disturbances. This method is superior to using a memory loop [11] because even spacing of the resonance modes is not a requirement, there is more freedom in the placement of the modes, and the number of resonance modes is a design option. In addition, the order or complexity of the internal model is typically smaller than the large number of delays required by the memory loop in the case of a high-frequency sampling.

4. IMB controllers

4.1. IMB property

The goal when using the internal model principle is to place closed loop transmission zeros at the location of the poles associated with the periodic disturbance. The following is the definition of an IMB controller.

Definition 1. (IMB controller) A strictly proper controller with a minimal state space realization $(A_c, B_c, C_c, 0)$ is IMB if the eigenvalues of A_c contain the eigenvalues of the internal model.

Here we consider only strictly proper controllers since we are in discrete time and require at least one step time delay between the input and output for calculations. This definition guarantees that the sensitivity function will have transmission zeros [19] at the proper location. The following Proposition 1 clarifies this point.

Proposition 1. *Consider the state space system given by (A, B, C, D) and a controller with a state space realization $(A_c, B_c, C_c, 0)$. If the controller is IMB then the output sensitivity function $S(q)$ will have transmission zeros located at the eigenvalues of the internal model.*

Proof. The output sensitivity function is defined as

$$S(q) := (I - P(q)C(q))^{-1}, \tag{7}$$

where

$$P(q) := \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \text{ is the plant and } C(q) := \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & 0 \end{array}\right] \text{ is the controller.}$$

The state space realization of $(I - P(q)C(q))$ is given by

$$(I - P(q)C(q)) = \left[\begin{array}{cc|c} A_c & B_cC & B_cD \\ 0 & A & B \\ \hline -C_c & 0 & I \end{array}\right]. \tag{8}$$

The inverse of the above is

$$(I - P(q)C(q))^{-1} = \left[\begin{array}{cc|c} A_c + B_cDC_c & B_cC & B_cD \\ BC_c & A & B \\ \hline C_c & 0 & I \end{array}\right] \tag{9}$$

and the trnasmission zeros are determined by the rank $(R(\lambda))$, where

$$R(\lambda) = \begin{bmatrix} A_c + B_cDC_c - \lambda I & B_cC & B_cD \\ BC_c & A - \lambda I & B \\ C_c & 0 & I \end{bmatrix} \tag{10}$$

and $\lambda \in \mathbb{C}$. Multiplying on the right with a full rank matrix gives

$$\begin{bmatrix} A_c + B_cDC_c - \lambda I & B_cC & B_cD \\ BC_c & A - \lambda I & B \\ C_c & 0 & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -C_c & 0 & I \end{bmatrix} = \begin{bmatrix} A_c - \lambda I & B_cC & B_cD \\ 0 & A - \lambda I & B \\ 0 & 0 & I \end{bmatrix}. \tag{11}$$

Therefore, the normal rank of $R(\lambda)$ drops when λ is an eigenvalue of A_c, and since A_c is IMB there are transmission zeros at the location of the eigenvalues of the internal model. □

The output sensitivity function is investigated because, for our problem, it represents the path from periodic disturbances to the output of the system and placing transmission zeros will reject these disturbances.

Many systems, such as the Active Noise Control problem presented in Section 2, experience periodic and non-periodic (random) disturbances. In addition to rejecting the periodic disturbances, it is desirable to suppress the random disturbances. Often there is a trade-off between the rejection of periodic and random disturbances just like the trade-off between minimizing the variance of the output and the variance of the control signal. The additional goal of minimizing the affects of the random disturbances can be handled by modifying standard LQG theory.

4.2. Optimal IMB control problem

Consider the state space models of the plant:

$$\begin{aligned} x_p(k+1) &= A_px_p(k) + B_uu(k) + B_ww(k), \\ y_p(k) &= C_px_p(k) + D_{yw}w(k) \end{aligned} \tag{12}$$

and the internal model

$$\begin{aligned} x_m(k+1) &= A_mx_m(k) + B_my_p(k), \\ y_m(k) &= C_mx_m(k) \end{aligned} \tag{13}$$

that will be used for the remainder of the paper. These systems are assumed to be linear discrete time systems, and $u(k)$ is the control signal, $w(k)$ is the random disturbance signal, $y_p(k)$ is the output of the plant that will be used for feedback, $x_p(k)$ is the plant states, and $x_m(k)$ are the internal model states. The disturbance $w(k)$ is assumed to be a zero mean Gaussian white noise sequence with unity covariance that is independent of the initial state vector, and models the non-periodic disturbances acting on the plant.

It is also convenient to use the following notation:

$$\left[\begin{array}{c|c} \mathscr{A} & \mathscr{B}_u \\ \hline \mathscr{C}_\zeta & \mathscr{D}_{zu} \end{array}\right] = \left[\begin{array}{cc|c} A_p & 0 & B_u \\ B_m C_p & A_m & 0 \\ \hline C_p & 0 & 0 \\ 0 & C_m & 0 \\ 0 & 0 & \alpha \end{array}\right]. \tag{14}$$

In Ref. [20] an IMB controller was derived by realizing that the internal model states are available for measurement and thus only the plant states need to be estimated. Using a state feedback gain that stabilizes the series connection of the plant with the internal model and a predictor of the plant states results in an IMB controller that can be written as

$$C_{IMB}(q) := \left[\begin{array}{cc|c} A_p - L_p C_p - B_u K_1 & -B_u K_2 & L_p \\ 0 & A_m & B_m \\ \hline -K_1 & -K_2 & 0 \end{array}\right], \tag{15}$$

where the gains $K = [K_1\, K_2]$ and L_p are chosen such that the closed loop system is stable. In the current paper, we seek the optimal gains that minimize the variance of a performance channel in the presence of random noise.

The optimal IMB control problem is to find the optimal gains K_1^*, K_2^*, and L_P^* for the controller given by Eq. (15) such that the cost

$$\begin{aligned} J &= \lim_{k\to\infty} E\left\{\zeta(k)^{\mathrm{T}}\zeta(k)\right\}, \\ &= \lim_{k\to\infty} E\left\{\left[x_p(k)^{\mathrm{T}} x_m(k)^{\mathrm{T}}\right]\mathscr{C}_\zeta^{\mathrm{T}}\mathscr{C}_\zeta \begin{bmatrix} x_p(k) \\ x_m(k) \end{bmatrix} + u(k)^{\mathrm{T}}\mathscr{D}_{zu}^{\mathrm{T}}\mathscr{D}_{zu}u(k)\right\} \end{aligned} \tag{16}$$

is minimized.

4.3. Optimal IMB controller

The optimal IMB controller is obtained by direct minimization of the cost function. In other papers, the design is handled by designing a stabilizing feedback gain of the series connection of the plant and internal model and a stable observer gain for the plant.

Theorem 1. (*Optimal IMB controller*) *Consider the plant given in Eq.* (12) *and the controller defined in Eq.* (15) *where the matrices A_m and B_m are a are a given. If* $(\mathscr{A}, \mathscr{B}_u)$ *and* (A_p, B_w) *are controllable* $(\mathscr{A}, \mathscr{C}_\zeta)$ and (A_p, C_p) *are observable*, $D_{yw}D_{yw}^{\mathrm{T}} > 0$ *and* $\mathscr{C}_{zu}^{\mathrm{T}}\mathscr{D}_{zu} = 0$, *then the control gains given by*

$$K^* = (\mathscr{B}_u^{\mathrm{T}} P_c \mathscr{B}_u + \mathscr{D}_{zu}^{\mathrm{T}}\mathscr{D}_{zu})^{-1}\mathscr{B}_u^{\mathrm{T}} P_c \mathscr{A} \tag{17}$$

and

$$L_p^* = (A_p P_o C_p^{\mathrm{T}} + B_w D_{yw}^{\mathrm{T}})(C_p P_o C_p^{\mathrm{T}} + R)^{-1}, \tag{18}$$

where P_c and P_o are the positive definite solutions to

$$P_c = \mathscr{A}^{\mathrm{T}} P_c \mathscr{A} + \mathscr{A}^{\mathrm{T}} P_c \mathscr{B}_u (\mathscr{B}_u^{\mathrm{T}} P_c \mathscr{B}_u + \mathscr{D}_{zu}^{\mathrm{T}}\mathscr{D}_{zu})^{-1}\mathscr{B}_u^{\mathrm{T}} P_s \mathscr{A} + \mathscr{C}_\zeta^{\mathrm{T}}\mathscr{C}_\zeta \tag{19}$$

and

$$P_o = A_p P_o A_p^{\mathrm{T}} - (B_w D_{yw}^{\mathrm{T}} + A_p P_o C_p^{\mathrm{T}})(C_p P_o C_p^{\mathrm{T}} + D_{yw} D_{yw}^{\mathrm{T}})^{-1}(C_p P_o A_p^{\mathrm{T}} + D_{yw} B_w^{\mathrm{T}}) + B_w B_w^{\mathrm{T}} \tag{20}$$

minimize the cost function defined in Eq. (3). Moreover, the minimum cost can be written as

$$J = tr\left\{C_p P_o C_p^{\mathrm{T}}\right\} + tr\left\{P_c \begin{bmatrix} L_p \\ B_m \end{bmatrix}\left(C_p P_o C_p^{\mathrm{T}} + D_{yw} D_{yw}^{\mathrm{T}}\right)\begin{bmatrix} L_p \\ B_m \end{bmatrix}^{\mathrm{T}}\right\}. \tag{21}$$

Proof. The closed loop system, with the performance channel $\zeta(k)$ as the output, can be written as

$$\left(\begin{array}{c} \tilde{x}(k+1) \\ \hat{x}(k+1) \\ x_m(k+1) \\ \hline \psi(k) \end{array}\right) = \left[\begin{array}{ccc|c} A_p - L_pC_p & 0 & 0 & B_w - L_pD_{yw} \\ L_pC_p & A_p - B_uK_1 & -B_uK_2 & L_pD_{yw} \\ B_mC_p & B_mC_p & A_m & B_mD_{yw} \\ \hline C_p & C_p & 0 & 0 \\ 0 & -\alpha K_1 & -\alpha K_2 & 0 \\ 0 & 0 & C_m & 0 \end{array}\right] \left(\begin{array}{c} \tilde{x}(k) \\ \hat{x}(k) \\ x_m(k) \\ w(k) \end{array}\right). \tag{22}$$

Define

$$L = \begin{bmatrix} L_p \\ B_m \end{bmatrix}, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} B_w - L_pD_{yw} \\ LD_{yw} \end{bmatrix} \tag{23}$$

and

$$\mathscr{X}_{k+1} = E\left\{ \begin{pmatrix} \tilde{x}(k+1) \\ \hat{x}(k+1) \\ x_m(k+1) \end{pmatrix} \begin{pmatrix} \tilde{x}(k+1) \\ \hat{x}(k+1) \\ x_m(k+1) \end{pmatrix}^{\mathrm{T}} \right\}, \tag{24}$$

then the covariance of the state satisfies

$$\mathscr{X}_{k+1} = \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix} \mathscr{X}_k \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} B_1B_1^{\mathrm{T}} & B_1B_2^{\mathrm{T}} \\ B_2B_1^{\mathrm{T}} & B_2B_2^{\mathrm{T}} \end{bmatrix}. \tag{25}$$

Because, the closed loop is stable, the limit as $k \to \infty$ exist and satisfies

$$\mathscr{X} = \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix} \mathscr{X} \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} B_1B_1^{\mathrm{T}} & B_1B_2^{\mathrm{T}} \\ B_2B_1^{\mathrm{T}} & B_2B_2^{\mathrm{T}} \end{bmatrix}. \tag{26}$$

Assuming that $\mathscr{X} = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ (to be proven later) gives

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} = \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix} \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} A_p - L_pC_p & 0 \\ LC_p & \mathscr{A} - \mathscr{B}_uK \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} B_1B_1^{\mathrm{T}} & B_1B_2^{\mathrm{T}} \\ B_2B_1^{\mathrm{T}} & B_2B_2^{\mathrm{T}} \end{bmatrix} \tag{27}$$

and the cost is given by

$$J = tr\left\{C_pP_1C_p^{\mathrm{T}}\right\} + tr\left\{ \begin{bmatrix} C_p & 0 \\ -\alpha K_1 & -\alpha K_2 \\ 0 & C_m \end{bmatrix} P_2 \begin{bmatrix} C_p^{\mathrm{T}} & -\alpha K_1^{\mathrm{T}} & 0 \\ 0 & -\alpha K_2^{\mathrm{T}} & C_m^{\mathrm{T}} \end{bmatrix} \right\}. \tag{28}$$

Multiplying the block matrix equation gives the following three equations:

$$P_1 = (A_p - L_pC_p)P_1(A_p - L_pC_p)^{\mathrm{T}} + (B_w - L_pD_{yw})(B_w - L_pD_{yw})^{\mathrm{T}}, \tag{29}$$

$$(A_p - L_pC_p)P_1C_p^{\mathrm{T}} - L_pD_{yw}D_{yw}^{\mathrm{T}} = 0, \tag{30}$$

$$P_2 = (\mathscr{A} - \mathscr{B}_uK)P_2(\mathscr{A} - \mathscr{B}_uK)^{\mathrm{T}} + L(C_pP_1C_p^{\mathrm{T}} + D_{yw}D_{yw}^{\mathrm{T}})L^{\mathrm{T}}. \tag{31}$$

It is known that if (A_p, B_w) is controllable, (A_p, C_p) is observable, and $D_{yw}D_{yw}^{\mathrm{T}} > 0$, then the gain

$$L_p^* = (A_pP_1C_p^{\mathrm{T}} + B_wD_{yw}^{\mathrm{T}})(C_pP_1C_p^{\mathrm{T}} + D_{yw}D_{yw}^{\mathrm{T}})^{-1} \tag{32}$$

is the optimal gain for Eq. (29). Using this gives

$$P_1 = A_pP_1A_p^{\mathrm{T}} - L_p^*(C_pP_1A_p^{\mathrm{T}} + D_{yw}B_w^{\mathrm{T}}) + B_wB_w^{\mathrm{T}}, \tag{33}$$

which equals Eq. (20).

Analyzing the dual form of Eq. (31) gives

$$P_2 = (\mathscr{A} - \mathscr{B}_u K)^{\mathrm{T}} P_2 (\mathscr{A} - \mathscr{B}_u K) + \begin{bmatrix} C_p^{\mathrm{T}} & -\alpha K_1^{\mathrm{T}} & 0 \\ 0 & -\alpha K_2^{\mathrm{T}} & C_m^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} C_p & 0 \\ -\alpha K_1 & -\alpha K_2 \\ 0 & C_m \end{bmatrix}, \tag{34}$$

where the cost becomes

$$\begin{aligned} J &= tr\{C_p P_1 C_p^{\mathrm{T}}\} + tr\{(C_p P_o C_p^{\mathrm{T}} + D_{yw} D_{yw}^{\mathrm{T}})^{1/2} L^{\mathrm{T}} P_2 L (C_p P_o C_p^{\mathrm{T}} + D_{yw} D_{yw}^{\mathrm{T}})^{1/2}\} \\ &= tr\{C_p P_1 C_p^{\mathrm{T}}\} + tr\{P_2 L (C_p P_o C_p^{\mathrm{T}} + D_{yw} D_{yw}^{\mathrm{T}}) L^{\mathrm{T}}\}. \end{aligned} \tag{35}$$

Eq. (34) can be rewritten as

$$P_2 = (\mathscr{A} - \mathscr{B}_u K)^{\mathrm{T}} P_2 (\mathscr{A} - \mathscr{B}_u K) + \mathscr{C}_\zeta^{\mathrm{T}} \mathscr{C}_\zeta + K^{\mathrm{T}} \mathscr{D}_{zu}^{\mathrm{T}} \mathscr{D}_{zu} K. \tag{36}$$

Using the fact that $\mathscr{C}_\zeta^{\mathrm{T}} \mathscr{D}_{zu} = 0$ and if $(\mathscr{A}, \mathscr{B}_u)$ is considerable,$(\mathscr{A}, \mathscr{C}_\zeta)$ is observable, and $\mathscr{D}_{zu}^{\mathrm{T}} \mathscr{D}_{zu} > 0$ then the optimal gain for Eq. (36) is $K^* = (\mathscr{B}_u^{\mathrm{T}} P_2 \mathscr{B}_u + \mathscr{D}_{zu}^{\mathrm{T}} \mathscr{D}_{zu})^{-1} \mathscr{B}_u^{\mathrm{T}} P_2 \mathscr{A}$. This gives Eq. (19). It is easily verified by direct substitution that Eq. (30) is satisfied. Therefore, all that remains to show is the block diagonal property of the state covariance.

Suppose there exists another positive definite covariance matrix $\tilde{\mathscr{X}}$ that is not block diagonal. First, after multiplying a full block covariance matrix $\tilde{\mathscr{X}} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix}$ in Eq. (26) reveals that P_{11} still must satisfy Eq. (20) to be optimal. Now assume that this new covariance matrix $\tilde{\mathscr{X}}$ is optimal, then we get

$$\begin{bmatrix} P_o & 0 \\ 0 & P_c \end{bmatrix} \geqslant \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix} \geqslant 0, \tag{37}$$

so that J is minimized. This gives that following condition:

$$\begin{bmatrix} P_o - P_{11} & -P_{12} \\ -P_{12}^{\mathrm{T}} & P_c - P_{22} \end{bmatrix} \geqslant 0 \tag{38}$$

and since P_o must equal P_{11}, this gives

$$\begin{bmatrix} 0 & -P_{12} \\ -P_{12}^{\mathrm{T}} & P_c - P_{22} \end{bmatrix} \geqslant 0, \tag{39}$$

which implies that $P_{12} = 0$ to preserve positive semidefiniteness. Therefore, the covariance matrix is block diagonal. $\square$

This Theorem 1 is the separation principle for IMB controllers and implies that the design of the optimal gains in an LQG design can be done independently. If the internal model is set to zero, then the usual separation principle is obtained. Thus, this is an extension of the usual separation principle for LQG controllers. It is necessary to weight the internal model states in the optimization otherwise there is an observability issue with $(\mathscr{A}, \mathscr{C}_\zeta)$. It is straightforward to derive similar results for continuous time systems. In addition, there are other parameterizations for IMB controllers that can lead to similar results. Another parametrization can be found in Ref. [13].

5. Real-time implementation of an IMB controller for noise cancellation

5.1. *Identification of plant and disturbance dynamics*

Models for the speaker/microphone dynamics and the non-periodic disturbances are obtained separately by using prediction-error methods [21] since the fan and the speaker can be controlled individually. First, a persistently exciting signal $u(k)$ is sent to the speaker while the fan is not running and the pick-up microphone

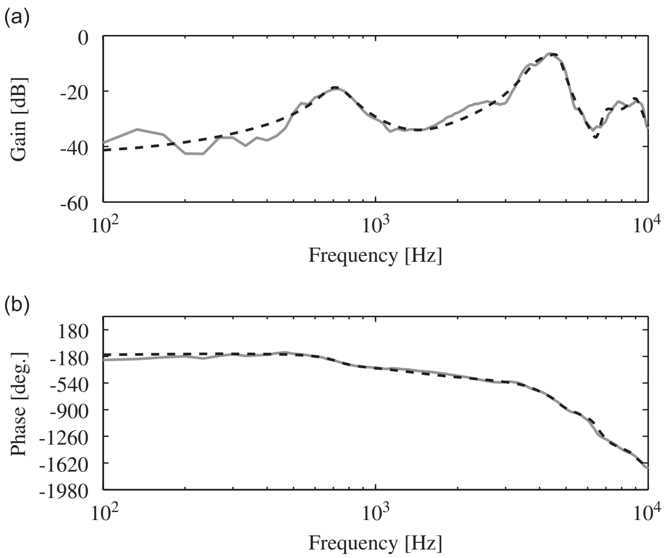

Fig. 2. Frequency response comparison between the parametrically identified output error model $G(q)$ and the spectral estimate of the acoustic system: (a) magnitude of frequency response and (b) phase of frequency response. Dashed line (black): $G(q)$. Solid line (gray): spectral estimate of the acoustic system.

signal $y_1(k)$ is recorded. These two signals ($u(k)$ and $y_1(k)$) are used to create an output error (OE) model [21] $G(q)$. Next, the fan is turned on while the speaker is turned off. The microphone signal during the second experiment $y_2(k)$ is recorded and an autoregressive (AR) model $A(q)^{-1}$ is used to match a filtered version of $y_2(k)$ so that the auto regressive model will have the same power spectrum as the non-periodic part of the fan noise. The non-periodic disturbances are added to the output of $G(q)$ to form the plant dynamic model.

The frequency response of the model $G(q)$ and the frequency response of the data generating system are shown in Fig. 2. The frequency response of the model matches the frequency response of the data generating system over the frequency range of interest. Accuracy in the low-frequency range is not needed and there is the low coherence between the $u(k)$ and $y_1(k)$ due to the inability of the speaker to generate sounds in this range. Accuracy is not desired at high frequencies because the response is small and the controller is not likely to have a high bandwidth.

The difference between $y_1(k)$ and the output of the OE model $G(q)$ with $u(k)$ as an input is shown in Fig. 3. It can be observed from the lower right of this figure that $G(q)$ captures the main resonance modes of the system, but does not capture some of the higher frequency dynamics. A higher order model that simulates the higher frequency dynamics more accurately could be used but the order of the controller will also increase. $G(q)$ was chosen with this size/accuracy trade-off in mind.

The magnitude squared of the frequency response of the AR model $|A(e^{j\omega})^{-1}|^2$ and the fan noise spectrum are shown in Fig. 4. The AR model fits the non-periodic part of the fan noise from 400 to 7000 Hz. Since the controller is designed to cancel the first four harmonics (at approximately 1000, 2000, 3000, and 4000 Hz) and the acoustic system is unable to generate low frequency noise, this model will be used in the control design.

5.2. Control design

The IMB control design method outlined in this paper was applied to the acoustic system described in Section 2. A pick-up microphone is used for feedback control of four miniature speakers that are mounted in close proximity, see Fig. 1. Acoustic noise from a server fan is attenuated with the IMB controller. Digital control is implemented in real-time with a Pentium based PC with a 12 bit signed AD/DA card.

The internal model that was used to design the IMB controller is shown in Fig. 5. The internal model has four distinct resonance modes located at 1000, 2000, 3000, and 4000 Hz. This internal model is undamped; this shows the effectiveness of this method to design marginally stable controllers, such as repetitive controllers.

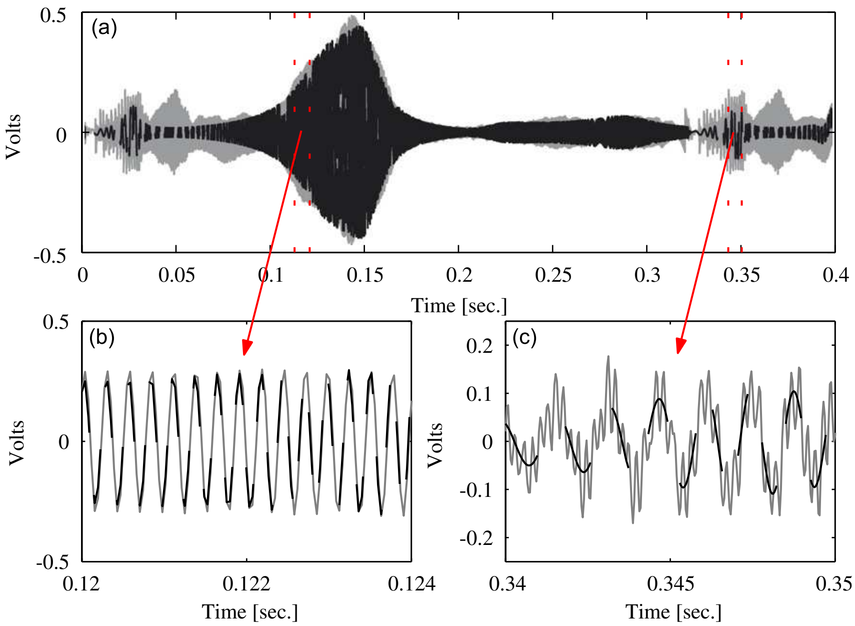

Fig. 3. Time series comparison between $G(q)$ and the data obtained from the acoustic system. (a) Time series data from 0 to 0.4 s. (b) Time series data from 0.120 to 0.124 s. (c) Time series data from 0.34 to 0.35 s. Dashed line (black): $G(q)$. Solid line (gray): acoustic system data.

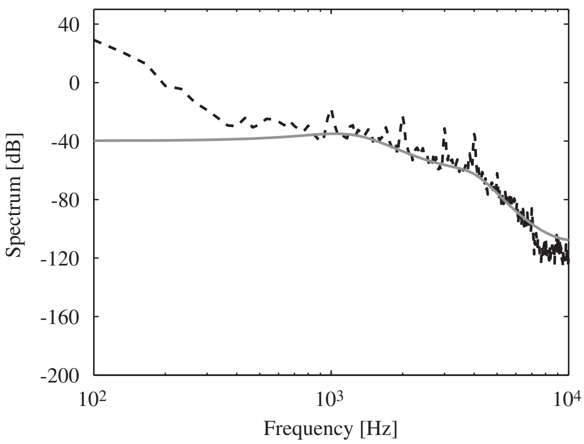

Fig. 4. $|A(e^{j\omega})^{-1}|^2$ and measured fan noise spectrum. Dashed line (black): power spectral density of fan noise. Solid line (gray): $|A(e^{j\omega})^{-1}|^2$.

The controller has a structure very similar to the internal model, see Figs. 5(a) and (b). The designer can shape the controller to given specifications and the design method will take care of performance and stability. In Fig. 5(a) is shown the Nyquist plot of the loop gain. The large gain is due to the marginally stable internal model and this is the designer's specification, whereas the angle at which the gain happens guarantees stability of the closed loop. This is an immediate result of the proposed design method.

5.3. Experimental results

Spectral analysis of the pick-up microphone signal before and after the IMB control is applied is shown in Fig. 6. In Fig. 6(a), the power spectrum of the pick-up microphone signal before IMB control is applied is shown. The harmonics of the server fan are very apparent. In Fig. 6(b), the power spectrum of the pick-up microphone during feedback is shown. Attenuation of the first four harmonics is achieved with this IMB

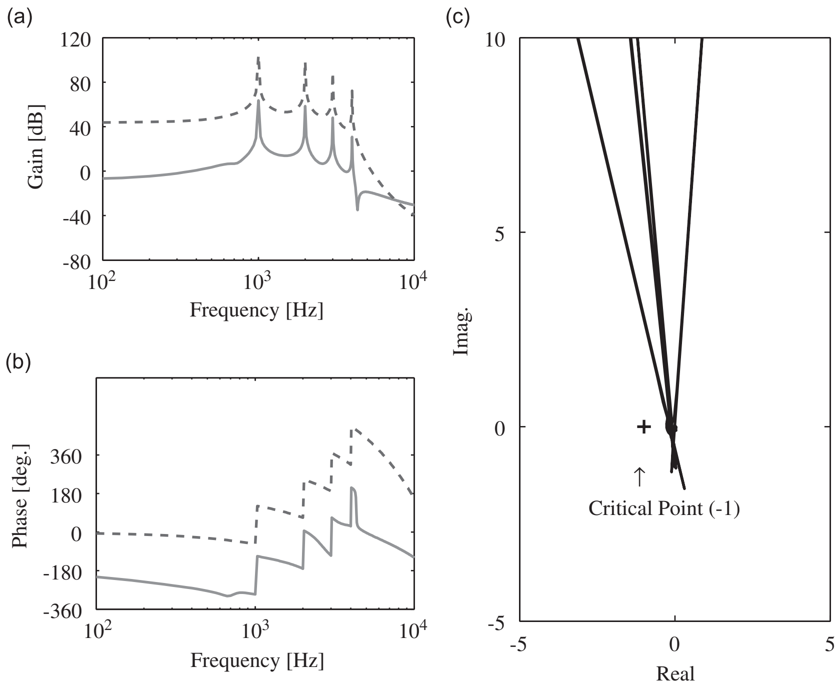

Fig. 5. IMB control results: (a) magnitude of the frequency response; (b) phase of the frequency response. Dashed line (black): internal model, solid line (gray): IMB controller; and (c) Nyquist plot of the loop gain.

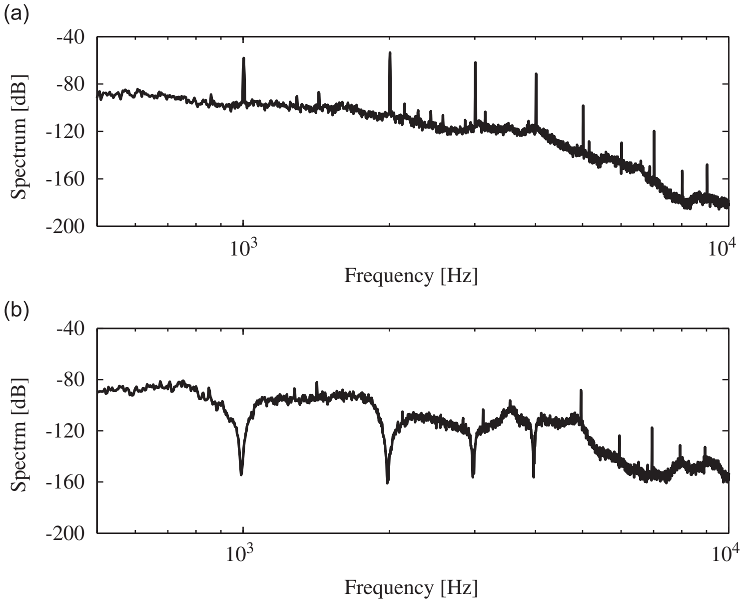

Fig. 6. Implementation results of the fan noise power spectrum: (a) spectrum without IMB control and (b) spectrum while IMB control is applied.

controller. Sound at neighboring frequencies is amplified a little bit due to Bode's sensitivity integral [19], also known as the water bed effect. However, this effect can be minimized by choosing the damping ratio of the internal model and the cost function wisely. No damping was used here to demonstrate the power and flexibility of this design method.

6. Conclusions

In this paper the design of an optimal, IMB controller was presented in the LQG control framework. It was shown that repetitive and IMB control problems can be solved in a optimal manner by modification of standard LQG theory.

The acoustic system used to demonstrate the effectiveness of the IMB controller consists of a pick-up microphone and four miniature speakers attached to the end of an acoustic duct. Noise from a server fan is attenuated with feedback control from the pick-up microphone to the miniature speakers. The power spectrum of the fan noise consists of periodic disturbances that appear as multiples of the fundamental frequency (due to the BPF) and non-periodic disturbances due to turbulence and vibrations.

An IMB controller was successfully designed and implemented in real-time for active noise cancelation. The IMB controller was able to attenuate the first four harmonics of the fan noise.

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