European Journal of Control,
Vol. 1, No.1, pp. 62-74, 1995

Identification of normalized coprime plant factors from closed loop experimental data [‡]

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Abstract

Recently introduced methods of iterative identification and control design are directed towards the design of high performing and robust control systems. These methods show the necessity of identifying approximate models from closed loop plant experiments. In this paper a method is proposed to approximately identify normalized coprime plant factors from closed loop data. The fact that *normalized* plant factors are estimated gives specific advantages both from an identification and from a robust control design point of view. It will be shown that the proposed method leads to identified models that are specifically accurate around the bandwidth of the closed loop system. The identification procedure fits very naturally into a recently developed the iterative identification/control design scheme based on $\mathcal{H}_\infty$ robustness optimization.

1 Introduction

Recently it has been motivated that the problem of designing a high performance control system for a plant with unknown dynamics through separate stages of (approximate) identification and model based control design requires iterative schemes

[‡]Report N-477, Mechanical Engineering Systems and Control Group, Delft University of Technology. Submitted for publication in the *European Journal of Control*, October 18, 1994.

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[♯]The work of Raymond de Callafon is sponsored by the Dutch "Systems and Control Theory Network".

to solve the problem [19, 20, 28, 31, 30, 41, 24]. In these iterative schemes each identification is based on new data collected while the plant is controlled by the latest compensator. Each new nominal model is used to design an improved compensator, which replaces the old compensator, in order to improve the performance of the controlled plant.

A few iterative schemes proposed in literature have been based on the prediction error identification method, together with LQG control design [41, 12, 14]. In [19, 20, 28, 29, 31] iterative schemes have been worked out, employing a Youla parametrization of the plant, and thus dealing with coprime factorizations in both identification and control design stage; as control design methods an $\mathcal{H}_\infty$ robustness optimization procedure of [23, 4] is applied in [28, 29, 31], while in [19, 20] the IMC-design method is employed. Alternatively, in [21] the identification and control design are based on covariance data. In [19] the IMC-design method is employed, and the identification step is replaced by a model reduction based on full plant knowledge. Alternatively, in [25] an iteration is used to build prefilters for a control-relevant prediction error identification from one open-loop dataset. For a general background and a more extensive overview and comparison of different iterative schemes the reader is referred to [10, 1, 36].

One of the central aspects in almost all iterative schemes is the fact that the identification of a control-relevant plant model has to be performed under closed loop experimental conditions. Standard identification methods have not been able to provide satisfactory models for plants operating in closed loop, except for the case that input/output dynamics and noise characterictics can be modelled exactly.

Recently introduced approaches to the closed loop identification problem [16, 27, 19, 28, 34, 36] show the possibility of also identifying approximate models, where the approximation criterion (if the number of data tends to infinity) becomes explicit, i.e. it becomes independent of the - unknown - noise disturbance on the data. This has opened the possibility to identify approximate models from closed loop data, where the approximation criterion explicitly can be "controlled" by the user, despite a lack of knowledge about the noise characteristics. In the corresponding iterative schemes of identification and control design this approximation criterion then is tuned to generate a control-relevant plant model. The identification methods considered in the iterative procedures presented in [28, 29, 19] employ a plant representation in terms of a coprime factorization $P = ND^{-1}$, while in [28, 29] the two plant factors N, D are separately identified from closed-loop data.

Coprime factor plant descriptions play an important role in control theory. The parametrization of the set of all controllers that stabilize a given plant greatly facilitates the design of controllers [39]. The special class of *normalized* coprime factorizations has its applications in design methods [23, 4] and robustness margins [38, 9, 26]. If we have only plant input-output data at our disposal, then a relevant question becomes how to model the normalized coprime plant factors as good as possible. In this

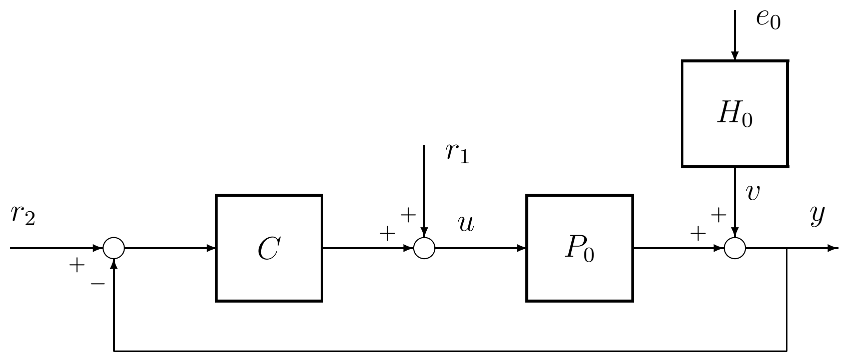

Fig. 1: Feedback configuration

paper we will focus on the problem of identifying normalized coprime plant factors on the basis of closed loop experimental data.

As an experimental situation we will consider the feedback configuration as depicted in Fig. 1, where P_0 is an LTI-(linear time-invariant), possibly unstable plant, H_0 a stable LTI disturbance filter, e_0 a sequence of identically distributed independent random variables and C an LTI-(possibly unstable) controller. The external signals r_1,r_2 can either be considered as external reference (setpoint) signals, or as (unmeasurable) disturbances. In general we will assume to have available only measurements of the input and output signals u and y, and knowledge of the controller C that has been implemented. We will also regularly refer to the artificial signal $r(t) := r_1(t)+Cr_2(t)$.

First we will discuss some preliminaries about normalized coprime factorizations and their relevance in control design. In section 3 a generalized framework is presented for closed loop identification of coprime factorizations. Next we present a multi-step algorithm for identification of normalized factors. In section 5 we will analyse the corresponding asymptotic identification criterion, and we will discuss the close relation with robustness margins in a gap-metric sense, being specifically relevant for the consecutive control design. In section 6 we will show the experimental results that were obtained when applying the identification algorithm to experimental data obtained from the radial servo-mechanism in a Compact Disc player.

$\mathcal{RH}_\infty$ will denote the set of real rational transfer functions in $\mathcal{H}_\infty$, analytic on and outside the unit circle; $\mathbb{R}[z^{-1}]$ is the ring of (finite degree) polynomials in the indeterminate z^{-1} and q is the forward shift operator: $qu(t) = u(t+1)$.

2 Preliminaries

Considering the feedback structure as depicted in figure 1 we will state that C stabilizes the plant P_0 if the mapping from $col(r_1, r_2)$ to $col(y, u)$ is stable, being equivalent to $T(P_0, C) \in \mathcal{RH}_\infty$ with

$$T(P_0, C) = \begin{bmatrix} P_0 \\ I \end{bmatrix} [I + CP_0]^{-1} \begin{bmatrix} C & I \end{bmatrix}. \tag{1}$$

Consider any LTI system P, then (following [39]) P has a *right coprime factorization* (r.c.f.) (N, D) over $\mathcal{RH}_\infty$ if there exist $U, V, N, D \in \mathcal{RH}_\infty$ such that

$$P(z) = N(z)D^{-1}(z); \quad UN + VD = I. \tag{2}$$

In addition a right coprime factorization (N_n, D_n) of P is called *normalized* if it satisfies

$$N_n^T(z^{-1})N_n(z) + D_n^T(z^{-1})D_n(z) = I. \tag{3}$$

Dual definitions exist for left coprime factorizations (l.c.f.).

One of the properties of normalized coprime factors is that they form a decomposition of the system P in minimal order (stable) factors. In other words, if the plant has McMillan degree n_p, then normalized coprime factors of P will also have McMillan degree n_p[1]. In the scalar case this implies that there will always exist polynomials $a, b, f \in \mathbb{R}[z^{-1}]$ of degree n_p such that $N_n = f(z^{-1})^{-1}b(z^{-1})$ and $D_n = f(z^{-1})^{-1}a(z^{-1})$.

In robust stability analysis normalized coprime factors play an important role in robustness issues with respect to several perturbation classes of systems. One of the important ones, is a perturbation class that is induced by the gap-metric [9]. This gap-metric between two (possibly non-stable) systems P_1, P_2 is defined as $\delta P_1, P_2) = max\{\vec{\delta}(P_1, P_2), \vec{\delta}(P_2, P_1)\}$, with

$$\vec{\delta}(P_1, P_2) := \inf_{U \in \mathcal{RH}_\infty} \left\| \begin{bmatrix} N_1 \\ D_1 \end{bmatrix} - \begin{bmatrix} N_2 \\ D_2 \end{bmatrix} U \right\|_\infty, \tag{4}$$

where (N_1, D_1), (N_2, D_2) are normalized r.c.f.'s of P_1, P_2 respectively.

The relevance of normalized coprime factors in robustness issues is illustrated in the following robustness result.

[1] In the exceptional case that P contains all-pass factors, (one of) the normalized coprime factors will have McMillan degree $< n_p$, see [33].

Proposition 2.1 ([9]) *Let $\hat{P}$ be a plant model that is stabilized by the controller C, and consider the following two classes of systems:*

$$\mathcal{P}_{gap}(\hat{P},\gamma) := \{P \mid \delta(P,\hat{P}) \leq \gamma\}, \tag{5}$$
$$\mathcal{P}_{dgap}(\hat{P},\gamma) := \{P \mid \vec{\delta}(P,\hat{P}) \leq \gamma\}. \tag{6}$$

Then for either of the two classes of systems, the result holds that a controller C will stabilize all elements if and only if $\|T(\hat{P},C)\|_\infty < \gamma^{-1}$.

This result shows that when we would have access to normalized coprime factors of a plant model, together with an error bound on these (estimated) factors (in the form of error bounds on the mismatches Δ_N and Δ_D), then immediate results follow for the robust stability of the plant.

On one hand, this result may not seem to be too striking, since a similar situation can be reached by any hard-bounded uncertainty on the system's transfer function, and application of the small gain theorem. However the ability to deal with unstable plants and the interpretation of the related uncertainty description in terms of the gap-metric are considered to be specific advantages. The latter aspect being motivated by the fact that closed loop properties of two systems will be close whenever their distance in terms of the gap metric is small.

The control design method of [4, 23] is directed towards optimizing this same robustness margin as discussed above. This control design method is characterized by

$$C = \arg\min_{\tilde{C}\in\mathcal{C}} \|V_1 T(\hat{P},\tilde{C})V_2\|_\infty \tag{7}$$

with V_1, V_2 user-chosen stable weighting functions and $\mathcal{C}$ an appropriate class of controllers considered. This control design is utilized in the iterative identification/control design scheme of [28, 29, 31].

3 Closed loop identification of coprime factorizations

3.1 Closed loop identification

The closed loop identification problem is not straighforwardly solvable in the prediction error framework in the case that one is not sure that exact models of the plant and its disturbances can be obtained in the form of a consistent estimate of P_0 and H_0. And even in this case results are restricted to the situation of a stable plant

P_0, [22]. What we would like to find - based on signal measurements - is a model $\hat{P}$ of a possibly unstable plant P_0 such that there exists an explicit approximation criterion $J(P_0, \hat{P})$ indicating the way in which P_0 has been approximated (at least asymptotically in the number of data), while $J(P_0, \hat{P})$ is independent of the unknown noise disturbance on the data.
Additionally one would like to be able to tune this approximation criterion to get an approximation of P_0 that is desirable in view of the control design to be performed. This explicit tuning of the approximation criterion is possible within the classical framework only when open-loop experiments can be performed.
Let's consider a few alternatives to deal with this closed-loop approximate identification problem, assuming the signal r is available from measurements[2]:

- If we know the controller C, we could do the following:
  Consider a parametrized model $P(\theta)$, $\theta \in \Theta$, and identify θ through:

  $$\varepsilon(t,\theta) = y(t) - \frac{P(\theta)}{1+P(\theta)C} r(t) \tag{8}$$

  by least squares minimization of the prediction error $\varepsilon(t,\theta)$.

  This first alternative leads to a complicatedly parametrized model set, and as a result it is not attractive, although it provides us with a consistent estimate of P irrespective of the noise modelling, and with an explicit approximation criterion. However attention has to be restricted to stable plants.

- Identify transfer functions

  $$H_{yr} = \frac{P}{1+PC} \quad \text{and} \quad H_{ur} = \frac{1}{1+PC}$$

  as black box transfer functions $\hat{H}_{yr}, \hat{H}_{ur}$, then an estimate of P can be obtained as $\hat{P} = \hat{H}_{yr}\hat{H}_{ur}^{-1}$.

  This method shows a decomposition of the problem in two parts, actually decomposing the system into two separate (high) order factors, sensitivity function and plant-times-sensitivity function. In this setting it will be hard to "control" the order of the model to be identified, as the quotient of the two estimated transfer functions $\hat{H}_{yr}$, $\hat{H}_{ur}$ will generally not cancel the common dynamics that are present in both functions. As a result the model order will become unnecessarily high.

[2]Similar results follow if either r_1 or r_2 are available from measurements.

- As a third alternative we can first identify H_{ur} as a black box transfer function $\hat{H}_{ur}$, and consecutively identify P from:

$$\varepsilon(t,\theta) = y(t) - P(\theta)\hat{u}_r(t) \quad \text{with } \hat{u}_r(t) := \hat{H}_{ur} r(t).$$

  This method is presented in [34]. It also uses a decomposition of the plant P in two factors as in the previous method, now requiring a very accurate estimate of H_{ur} in the first step. An explicit approximation criterion can be formulated.

If, as in the last two methods, the plant is represented as a quotient of two factors of which estimates can be obtained from data, it is advantageous to let these factors have the minimal order, thus avoiding the problem that the resulting plant model has an excessive order, caused by non-cancelling terms with redundant dynamics. This will be discussed in the next subsection.

3.2 A generalized framework

We will now present a generalized framework for identification of coprime plant factors from closed loop data, allowing the situation to identify unstable models for unstable plants. It will be shown to have close connections to the Youla-parametrization, as employed in the identification schemes as proposed in [16, 27, 28, 19].
Let us consider the notation[3]

$$\begin{aligned} S_0(z) &= (I + C(z)P_0(z))^{-1} \quad \text{and} && (9)\\ W_0(z) &= (I + P_0(z)C(z))^{-1}. && (10) \end{aligned}$$

Then we can write the system's equations as[4]

$$\begin{aligned} y(t) &= P_0(q)S_0(q)r(t) + W_0(q)H_0(q)e_0(t) && (11)\\ u(t) &= S_0(q)r(t) - C(q)W_0(q)H_0(q)e_0(t). && (12) \end{aligned}$$

Note also that

$$r(t) = r_1(t) + C(q)r_2(t) = u(t) + C(q)y(t). \tag{13}$$

Using knowledge of $C(q)$, together with measurements of u and y, we can simply "reconstruct" the reference signal r in (13). So in stead of a measurable signal r, we can equally well deal with the situation that y, u are measurable and C is known.

[3]The main part of the paper is directed towards multivariable systems, and so we distinguish between output and input sensitivity.

[4]Note that we have employed the relations $W_0P_0 = P_0S_0$ and $S_0C = CW_0$.

It can easily be verified from (11),(12) that the signal $\{u(t) + C(q)y(t)\}$ is uncorrelated with $\{e_0(t)\}$ provided that r is uncorrelated with e_0. This shows with equations (11),(12) that the identification problem of identifying the transfer function from signal r to $col(y, u)$ is an "open loop"-type of identification problem, since r is uncorrelated with the noise terms dependent on e_0. The corresponding factorization of P_0 that can be estimated in this way is the factorization (P_0S_0, S_0), i.e. $P_0 = (P_0S_0)\cdot S_0^{-1}$, as also employed in e.g. [42].
However this is only one of the many factorizations that can be identified from closed loop data in this way. By introducing an auxiliary signal

$$x(t) := F(q)r(t) = F(q)(u(t) + C(q)y(t)) \tag{14}$$

with $F(z)$ a fixed stable rational transfer function, we can rewrite the system's relations as

$$y(t) = P_0(q)S_0(q)F(q)^{-1}x(t) + W_0(q)H_0(q)e_0(t) \tag{15}$$
$$u(t) = S_0(q)F(q)^{-1}x(t) - C(q)W_0(q)H_0(q)e_0(t), \tag{16}$$

and thus we have obtained another factorization of P_0 in terms of the factors $(P_0S_0F^{-1}, S_0F^{-1})$. Since we can reconstruct the signal x from measurement data, these factors can also be identified from data, as in the situation considered above, provided of course that the factors themselves are stable. We will now characterize the freedom that is present in choosing this filter F.

Proposition 3.1 *Consider a data generating system according to* (11),(12)*, such that C stabilizes P_0, and let $F(z)$ be a rational transfer function defining*

$$x(t) = F(q)(u(t) + C(q)y(t)). \tag{17}$$

Let the controller C have a left coprime factorization $(\tilde{D}_c, \tilde{N}_c)$. Then the following two expressions are equivalent

a. the mappings $col(y, u) \to x$ and $x \to col(y, u)$ are stable;

b. $F(z) = W\tilde{D}_c$ with W any stable and stably invertible rational transfer function.

The proof of this Proposition is added in the appendix.
Note that stability of the mappings mentioned under (a) is required in order to guarantee that we obtain a bounded signal x as an input in our identification procedure, and that the factors to be estimated are stable, so we are able to apply the standard (open-loop) prediction error methods and analysis thereof.

Note also that all factorizations of P_0 that are induced by these different choices of F reflect factorizations of which the stable factors can be identified from input/output data, cf. equations (15),(16).
The construction of the signal x is schematically depicted in Figure 2. Here we have employed (13) which clearly shows that x is uncorrelated with e_0 provided the external signals are also uncorrelated with e_0.

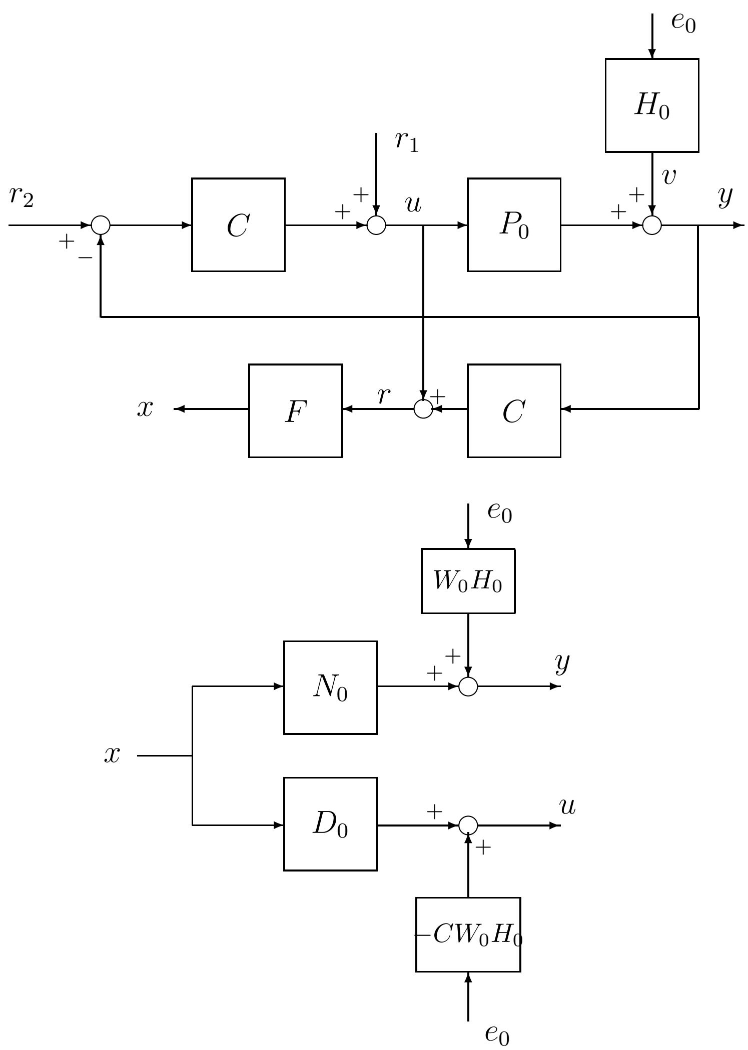

Fig. 2: Identification of coprime factors from closed loop data.

For any choice of F satisfying the conditions of Proposition 3.1 the induced factorization of P_0 is right coprime, as shown next.

Proposition 3.2 *Consider the situation of Proposition 3.1. For any choice of* $F = W\tilde{D}_c$ *with* W *stable and stably invertible, the induced factorization of* P_0*, given by* $(P_0S_0F^{-1}, S_0F^{-1})$ *is right coprime.* □

Proof: Let (X, Y) be right Bezout factors of (N, D), i.e. $XN + YD = I$, and denote $[X_1\ Y_1] = W(\tilde{D}_c D + \tilde{N}_c N)[X\ Y]$. Then by employing (A.1) it can simply be verified that X_1, Y_1 are stable and satisfy $X_1 P_0 S_0 F^{-1} + Y_1 S_0 F^{-1} = I$. □

We will exploit the freedom in the filter F, in order to tune the specific coprime factors that can be estimated from closed loop data.
However become we start the discussion about the specifici choice of F we present an alternative formulation for the freedom that is present in this choice of F, as formulated in the following Proposition.

Proposition 3.3 *The filter F yields stable mappings $(y, u) \to x$ and $x \to (y, u)$ if and only if there exists an auxiliary system P_x with rcf (N_x, D_x), stabilized by C, such that $F = (D_x + CN_x)^{-1}$. For all such F the induced factorization $P_0 = N_0 D_0^{-1}$ is right coprime.*

Proof: Consider the situation of Proposition 3.1. First we show that for any C with lcf $\tilde{D}_c^{-1}\tilde{N}_c$ and any stable and stably invertible W there always exists a system P_x with rcf $N_x D_x^{-1}$, being stabilized by C, such that $W = [\tilde{D}_c D_x + \tilde{N}_c N_x]^{-1}$.
Take a system P_a with rcf $N_a D_a^{-1}$ that is stabilized by C. With Lemma A.1 it follows that $\tilde{D}_c D_a + \tilde{N}_c N_a = \Lambda$ with Λ stable and stably invertible. Then choosing $D_x = D_a \Lambda^{-1} W^{-1}$ and $N_x = N_a \Lambda^{-1} W^{-1}$ delivers the desired rcf of a system P_x as mentioned above.
Since $F = W\tilde{D}_c$ and substituting $W = [\tilde{D}_c D_x + \tilde{N}_c N_x]^{-1}$ it follows that $F = [D_x + CN_x]^{-1}$. □

Employing this specific characterization of F, the coprime plant factors that can be identified from closed loop data satisfy

$$\begin{pmatrix} N_0 \\ D_0 \end{pmatrix} = \begin{pmatrix} P_0(I + CP_0)^{-1}(I + CP_x)D_x \\ (I + CP_0)^{-1}(I + CP_x)D_x \end{pmatrix}. \tag{18}$$

Now the auxiliary system P_x and its coprime factorization can act as a design variable that can be chosen so as to reduce the redundant dynamics in both coprime factors N_0, D_0.
In the next section we will show how we can exploit the freedom in choosing F, N_x and D_x in order to arrive at an estimate of *normalized* coprime factors of the plant.

The representation of P_0 in terms of the coprime factorization above, shows great resemblance with the dual Youla-parametrization [16, 19, 27], i.e. the parametrization of all plants that are stabilized by a given controller, as reflected in the following proposition.

Proposition 3.4 *Let C be a controller with rcf (N_c, D_c), and let P_x with rcf (N_x, D_x) be any system that is stabilized by C. Then*

(a) A plant P_0 is stabilized by C if and only if there exists an $R \in \mathcal{RH}_\infty$ such that

$$P_0 = (N_x + D_c R)(D_x - N_c R)^{-1} \tag{19}$$

is a right coprime factorization of P_0.

(b) For any such P_0, the corresponding stable transfer function R in (19) is uniquely determined by

$$R = D_c^{-1}(I + P_0 C)^{-1}(P_0 - P_x)D_x. \tag{20}$$

(c) The coprime factorization in (19) is uniquely determined by

$$N_x + D_c R = P_0(I + CP_0)^{-1}(I + CP_x)D_x \tag{21}$$

$$D_x - N_c R = (I + CP_0)^{-1}(I + CP_x)D_x \tag{22}$$

Proof: The proof of part (a), which actually boils down to the Youla-parametrization, is given in [6].
Part (b). With (19) it follows that $P_0[D_x - N_c R] = N_x + D_c R$. This is equivalent to $[D_c + P_0 N_c]R = P_0 D_x - N_x$ which in turn is equivalent to $[I + P_0 C]D_c R = [P_0 - P_x]D_x$ which proves the result.
Part (c). Simply substituting the expression (20) for R shows that

$$\begin{bmatrix} N_0 \\ D_0 \end{bmatrix} := \begin{bmatrix} N_x + D_c R \\ D_x - N_c R \end{bmatrix} = \begin{bmatrix} N_x + (I + P_0 C)^{-1}(P_0 - P_x)D_x \\ D_x - C(I + P_0 C)^{-1}(P_0 - P_x)D_x \end{bmatrix} \tag{23}$$

$$= \begin{bmatrix} P_x + (I + P_0 C)^{-1}(P_0 - P_x) \\ I - C(I + P_0 C)^{-1}(P_0 - P_x) \end{bmatrix} D_x \tag{24}$$

which proves the result, employing the relations $C(I + P_0 C)^{-1} = (I + CP_0)^{-1}C$ and $(I + P_0 C)^{-1}P_0 = P_0(I + CP_0)^{-1}$. □

This result shows that the coprime factorization that is used in the Youla parametrization is exactly the same coprime factorization that we have constructed in the previous section, by exploiting the freedom in the prefilter F, see Proposition 3.3.

4 An algorithm for approximate identification of normalized coprime factors

In order to obtain an approximation of a normalized coprime factorization of the unknown plant P_0 on the basis of closed loop experiments, similar as in [34], a procedure built up from two step, will be employed. These two steps can be formulated as follows.

1. In the first step of the procedure, the coprime factors $(N_{0,F}, D_{0,F})$ of (18) that are accessible from closed loop data, will be shaped in such a way that $(N_{0,F}, D_{0,F})$ becomes (almost) normalized. The rationale behind this idea is induced by the fact that the shape of the factorization $(N_{0,F}, D_{0,F})$ depends on the specific factorization (N_x, D_x) of the auxiliary model $P_x = N_x D_x^{-1}$ used in the filter F, see (18). From the ideal situation wherein the auxiliary model P_x satisfies $P_x = P_0$, it follows $D_{0,F} = D_x$ and $N_{0,F} = P_0 D_x = N_x$ from (18). Consequently, the normalization of $(N_{0,F}, D_{0,F})$ can be approached by letting P_x to be an accurate (inevitably high order) approximation of the plant P_0 and factorizing P_x in a normalized *rcf* (N_x, D_x). In order to obtain such an accurate auxiliary model P_x, the following algorithm based on coprime factor estimation can be proposed. A formal justification and analysis is postponed until the next section.

   To initialize the algorithm, consider an auxiliary model P_x that is internally stabilized by the known controller C. Now construct a normalized *rcf* (N_x, D_x) of the auxiliary model P_x. Procedures to compute a normalized *rcf* can for example be found in [40] for continues time systems and [3] for discrete time systems. Using this normalized *rcf* (N_x, D_x), compute the data filter

   $$F(q) = (D_x(q) + C(q)N_x(q))^{-1}$$

   according to Proposition 3.3 and simulate the auxiliary input

   $$x(t) = F(q) \begin{bmatrix} C(q) & I \end{bmatrix} \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}$$

   to be used for the identification. After this initialization, the algorithm reads as follows.

   1.a Use the signals $x(t)$ and $[y(t)\ u(t)]^T$ in a (least squares) identification algorithm with an output error model structure, minimizing $\|\varepsilon(t, \theta)\|_2$ over

θ, with

$$\varepsilon(t,\theta) = \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} - \begin{bmatrix} N(q,\theta) \\ D(q,\theta) \end{bmatrix} x(t) \tag{25}$$

considering $[y(t)\ u(t)]^T$ as output signal and $x(t)$ as input signal. The factorization $(N(q,\theta), D(q,\theta))$ being estimated here, will be used only to update and improve the auxiliary model P_x. Therefore, high-order modelling employing a parametrization based on orthonormal basis functions in a linear regression scheme will be used. In this respect, the new method of constructing orthogonal basis functions that contain system dynamics, see [18, 37], has shown promising results for identification purposes [5].

1.b Compute the model $P(q,\theta)$ on the basis of the factorization $(N(q,\theta), D(q,\theta))$ being estimated by

$$P(q,\theta) = N(q,\theta)D(q,\theta)^{-1}$$

and update the auxiliary model simply by $P_x(q) = P(q,\theta)$

1.c Again construct a normalized *rcf* (N_x, D_x) of the auxiliary model P_x, compute the data filter

$$F(q) = (D_x(q) + C(q)N_x(q))^{-1}$$

according to Proposition 3.3 and simulate the auxiliary input

$$x(t) = F(q) \begin{bmatrix} C(q) & I \end{bmatrix} \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}.$$

If the auxiliary model is satisfactory, the factorization $(N_{0,F}, D_{0,F})$, which is the relation between the simulated auxiliary signal $x(t)$ and the signal $[y(t)\ u(t)]^T$, will be (almost) normalized and the second step of the procedure can be invoked. Otherwise the steps 1.a till 1.c may be repeated to improve the quality of the auxiliary model P_x.

2. In the second step of the procedure, the simulated auxiliary signal $x(t)$, coming from the first step, and the signal $[y(t)\ u(t)]^T$ is used to perform an approximate identification of the (almost) normalized factorization $(N_{0,F}, D_{0,F})$ again using an output error structure, where $(N(q,\theta), D(q,\theta))$ can be parametrized as

$$\begin{bmatrix} N(q,\theta) \\ D(q,\theta) \end{bmatrix} = \begin{bmatrix} B_N(q,\theta) \\ B_D(q,\theta) \end{bmatrix} A(q,\theta)^{-1}$$

where B_N, B_D and A are polynominal (matrices) of proper dimensions. In this parametrization, where $N(q,\theta)$ and $D(q,\theta)$ have a common right divisor $A(q,\theta)$, guarantees that the McMillan degree of the model $P(q,\theta) = N(q,\theta)D^{-1}(q,\theta)$ is determined by the polynominal matrices $B_N(q,\theta)$ and $B_D(q,\theta)$ only.

The parameter estimate is obtained by a least squares minimization [22] of the filtered prediction error $\varepsilon_f(t,\theta)$ with $\varepsilon_f(t,\theta) = L\varepsilon(t,\theta)$, and $L \in \mathcal{RH}_\infty^{(n_y+n_u)\times(n_y+n_u)}$, decomposed as $L = diag(L_y, L_u)$.

The result of the procedure proposed above is composed of a (possibly low order) approximation $(N(q,\hat{\theta}), D(q,\hat{\theta}))$ and resulting model $P(q,\hat{\theta}) = N(q,\hat{\theta})D^{-1}(q,\hat{\theta})$ of an (almost) normalized right coprime factorization $(N_{0,F}, D_{0,F})$ of the plant P_0. It should be noted that the coprime factorizations $(N_{0,F}, D_{0,F})$ that can be accessed in this procedure can be made exactly normalized only in the situation that we have exact knowledge of the plant P_0. In the procedure presented above, this exact knowledge of P_0 has been replaced by a (very) high order accurate estimate of P_0. Note that the order of the "high order" estimate of $P(q,\theta)$ in step 1.b may be strongly dependent on the auxiliary model P_x that is used in the filter F to construct the auxiliary signal $x(t)$. The more accurate this auxiliary model, the more common dynamics is cancelled in the coprime factors (18), and consequently the easier the factorization $(N_{0,F}, D_{0,F})$ can be accurately described by a model of limited order. This highly motivates the usage of an iterative repetition of steps 1.a till 1.c in the algorithm presented above. Such an iterative procedure has also been applied in the application example discussed in section 6.

5 Analysis of the algorithm

In order to explicitly write down the asymptotic identification criterion that has been used in the final step of the algorithm, we write the related prediction error as

$$\varepsilon(t,\theta) = \begin{bmatrix} L_y & 0 \\ 0 & L_u \end{bmatrix} \left\{ \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} - \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} x(t) \right\} \tag{26}$$

$$= \begin{bmatrix} L_y & 0 \\ 0 & L_u \end{bmatrix} \left\{ \begin{bmatrix} N_0 - N(\theta) \\ D_0 - D(\theta) \end{bmatrix} x(t) + \begin{bmatrix} W_0 H_0 \\ -C W_0 H_0 \end{bmatrix} e_0(t) \right\} \tag{27}$$

with N_0, D_0 given by (24). As a result the asymptotic parameter estimate $\theta^* = \text{plim}_{N\to\infty} \hat{\theta}_N$ is characterized by

$$\theta^* = \arg \min_\theta \int_{-\pi}^{\pi} \Big[|N_0(e^{i\omega}) - N(e^{i\omega},\theta)|^2 |L_y(e^{i\omega})|^2 +$$

$$+ |D_0(e^{i\omega}) - D(e^{i\omega},\theta)|^2 |L_u(e^{i\omega})|^2\Big] \cdot \Phi_x(\omega) d\omega \tag{28}$$

with $x(t) = D_x^{-1}(I + CP_x)^{-1}[u(t) + C(q)y(t)]$.
We will write this expression as

$$\theta^* = \arg\min_\theta \left\| \begin{bmatrix} L_y[N_0 - N(\theta)] \\ L_u[D_0 - D(\theta)] \end{bmatrix} H_x \right\|_2 \tag{29}$$

where H_x is the monic stable spectral factor of Φ_x.
We will now write this identification criterion in terms of (exact) normalized coprime factors of the considered plant.

Proposition 5.1 *Consider the plant factors N_0, D_0 given by (24) to be identified from data as in the final step of the algorithm presented before, employing an auxiliary system $P_x = N_x D_x^{-1}$ stabilized by C, with N_x, D_x a normalized rcf.*
Using an output error model structure to identify N_0, D_0, as denoted in (25) with a least squares identification criterion (29), the asymptotic parameter estimate θ^* will satisfy

$$\theta^* = \arg\min_\theta \left\| \begin{bmatrix} L_y[N_{0,n}Q - N(\theta)] \\ L_u[D_{0,n}Q - D(\theta)] \end{bmatrix} H_x \right\|_2 \tag{30}$$

with $N_{0,n}, D_{0,n}$ a normalized rcf of the plant P_0 and Q the unique monic, stable and stably invertible solution to

$$Q^*Q = R^*FR + R^*G + G^*R + I \tag{31}$$

with

$$\begin{aligned} F &= D_c^* D_c + N_c^* N_c & (32) \\ G &= D_C^* N_x - N_c^* D_x. & (33) \end{aligned}$$

Proof: Using the expressions (24) for N_0 and D_0 it follows that $D_0^* D_0 + N_0^* N_0 = R^*FR + R^*G + G^*R + I$. Since this expression is positive real, there exists a unique Q, with $Q, Q^{-1} \in \mathcal{RH}_\infty$ and Q monic satisfying (31). As a result it follows that $(N_0 Q^{-1}, D_0 Q^{-1})$ is a normalized rcf of P_0. □

If the first identification step (Steps 1-3) of identifying (N, D) is accurately enough ($P_n \to P_0$), then in the second step of the procedure (Steps 4-6) , $P_x \to P_0$, with N_x, D_x a normalized rcf of P_x. As $P_x \to P_0$, applying (20) shows that $R \to 0$, and the R-dependent terms in (24) will vanish in the second identification step (Steps 4-6). In terms of the matrix Q as used in the expression (30) this shows as follows.

Proposition 5.2 *Consider the situation of Proposition 5.1. Then*

(a) $\|Q-I\|_\infty \to 0$ *as* $\|P_x - P_0\|_\infty \to 0.$

(b) $\|Q^{-1}-I\|_\infty \to 0$ *as* $\|P_x - P_0\|_\infty \to 0.$

Proof: Note that $\|Q^*Q - I\|_\infty = \|R^*AR + R^*B + B^*R\|_\infty$. For $\|P_x - P_0\|_\infty \to 0$ it follows with (20) that $\|R\|_\infty \to 0$ and so $\|Q^*Q - I\|_\infty \to 0$.
If $\|Q^*Q - I\|_\infty = 0$, and using the restriction that $Q, Q^{-1} \in \mathcal{RH}_\infty$ and Q monic, it implies that $Q = I$.
Using continuity properties of Q as a function of R the result follows. □

Our result now shows a similar type of expression as in the original two-step method of [34], with an approximation criterion in identification (30) that becomes very nice in case $P_x = P_0$, and consequently $Q = I$, but that also shows the deviation of the desired criterion as a result of a non-perfect first step.

Finally we will show the relation of the asymptotic identification criterion with a specific upper bound for a directed gap metric measure, which has direct implications for robust stability properties of a controller to be designed on the basis of the identified model.
For simplicity of notation and without loss of generality we will restrict attention to the situation $L_y = L_u = H_x = I$.
We take as a starting point that our identification that has resulted in (30) provides us with an $\alpha \in \mathbb{R}$ satisfying

$$\left\| \begin{bmatrix} N_0 - N(\theta^*) \\ D_0 - D(\theta^*) \end{bmatrix} \right\|_\infty \leq \alpha \tag{34}$$

For the construction of this α one can apply an alternative identification procedure that provides direct expressions and sometimes even minimization of the $\mathcal{RH}_\infty$-error in (30), see e.g. [17, 11, 13] for an approach in a worst-case deterministic setting, and [7, 8, 15] for approaches that incorporate probabilistic aspects.
If we have this α avialable we can apply the following result.

Proposition 5.3 *Consider the identification setup discussed before, and suppose that we have available an expression*

$$\| \begin{bmatrix} N_0 \\ D_0 \end{bmatrix} - \begin{bmatrix} N(\theta^*) \\ D(\theta^*) \end{bmatrix} \|_\infty \leq \alpha. \tag{35}$$

Then $\vec{\delta}(P_0, P(\theta^*)) \leq \alpha \|Q^{-1}\|_\infty$ *with* Q *defined as before.*

Proof: Combining (34) and (30) it follows that

$$\left\| \begin{bmatrix} N_{0,n} \\ D_{0,n} \end{bmatrix} Q - \begin{bmatrix} N(\theta^*) \\ D(\theta^*) \end{bmatrix} \right\|_\infty \|Q^{-1}\|_\infty \leq \alpha \|Q^{-1}\|_\infty \tag{36}$$

which leads us to

$$\left\| \begin{bmatrix} N_{0,n} \\ D_{0,n} \end{bmatrix} - \begin{bmatrix} N(\theta^*) \\ D(\theta^*) \end{bmatrix} Q^{-1} \right\|_\infty \leq \alpha \|Q^{-1}\|_\infty. \tag{37}$$

Using the definition of the directed gap-metric now shows the result. □

In terms of control design, on the basis of this model that is identified, the following interesting result can now be formulated.

Proposition 5.4 *Consider the control design scheme as discussed in* [23, 2]*, characterized by*

$$C_{\hat{P}} = \arg \min_{\tilde{C} \in \mathcal{C}} \|T(\hat{P}, \tilde{C})\|_\infty \tag{38}$$

with $\hat{P} = P(\theta^*)$*. If this controller satisfies*

$$\|T(\hat{P}, C_{\hat{P}})\|_\infty \leq \frac{1}{\alpha \|Q^{-1}\|_\infty} \tag{39}$$

then the plant P_0 *will be stabilized by* $C_{\hat{P}}$*.*

Proof: Follows directly using the robust stability results for uncertainty sets in the directed gap metric, see Proposition 2.1. □

This proposition shows that we can test a priori whether the designed controller will stabilize our plant, before actually implementing it. To this end we need an expression for (an upper bound on) the $\mathcal{RH}_\infty$-error that is made in the identification of coprime factors, actually in both steps of the identification procedure.

Note that in this procedure there is no need to use a parametrization of the model $P(\theta)$ in terms of normalized coprime factorizations. We have chosen the auxiliary system in such a way that the *plant* factors that are identified are almost normalized, and this is sufficient to obtain the given results reflecting robust stability properties.

6 Application to a mechanical servo system

We will illustrate the proposed identification algorithm by applying it to real life data obtained from experiments on the servo mechanism of a radial control loop, present in a compact disc player. The radial servo mechanism uses a radial actuator which consists of a permanent magnet/coil system mounted on the radial arm, in order to position the laser spot orthogonally to the tracks of the compact disc. For a more extensive description of this servo mechanism, we refer to [5, 32].

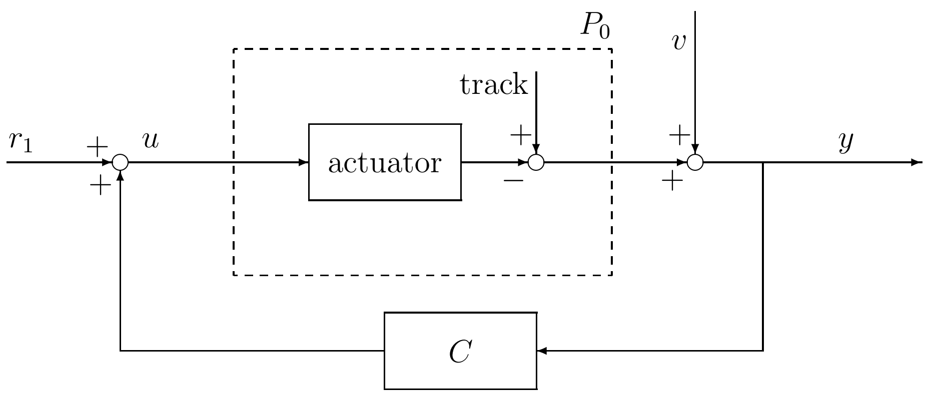

Fig. 3: Block diagram of the radial conrtol loop

A simplified representation of the experimental set up of the radial control loop is depicted in Figure 3, wherein P_0 and C denote respectively the radial actuator and the known controller. The radial servo mechanism is marginally stable, due to the presence of a double integrator in the radial actuator P_0. This experimental set up is used to gather time sequences of 8192 points of the input $u(t)$ to the radial actuator P_0 and the disturbed track position error $y(t)$ in closed loop, while exciting the control loop only by a bandlimited (100Hz–10kHz) white noise signal $r_1(t)$, added on the input $u(t)$.

The results of applying the two steps of the procedure presented in section 4 are shown in a couple of figures. Recall from section 4, that in the first step the aim is to find an auxiliary model P_x with a normalized *rcf* (N_x, D_x) used in the filter F, such that the factorization $(N_{0,F}, D_{0,F})$ of the actuator P_0 becomes (almost) normalized. This effect has been illustrated in Figure 4, where the result of the first step has been depicted.

In Figure 4(a) an amplitude plot of a spectral estimate of the factors $N_{0,F}$ and $D_{0,F}$, respectively denoted by $\hat{N}_{0,F}$ and $\hat{D}_{0,F}$, along with the factorization N_x and D_x of a high (24th) order auxiliary model P_x has been plotted. The result has been obtained by running through the steps 1.a – 1.c only three times. Additionally, in Figure 4(b) an evaluation of $N_{0,F}{}^* N_{0,F} + D_{0,F}{}^* D_{0,F}$ using the spectral estimates $\hat{N}_{0,F}$ and $\hat{D}_{0,F}$ is plotted to illustrate the fact that indeed the factorization $(N_{0,F}, D_{0,F})$ of the actuator P_0 where we have access to, trough the signals $x(t)$ and $[y(t)\ u(t)]^T$ is (almost) normalized.

Finally, Figure 5 presents the result of a low (10th) order approximation of the factorization $(N_{0,F}, D_{0,F})$ of the actuator P_0, which is the second step in the procedure mentioned in section 4. In Figure 5(a) an amplitude plot of the obtained factorization $(N(\hat{\theta}), D(\hat{\theta}))$ along with the spectral estimate of the factorization $(N_{0,F}, D_{0,F})$ has been draw. To be complete, an amplitude plot of the finally obtained 10th order model $P(\hat{\theta}) = N(\hat{\theta})D^{-1}(\hat{\theta})$ along with a spectral estimate has been depicted in Figure 6(b).

Conclusions

In this paper it is shown that it is possible to identify (almost) normalized coprime plant factors based on closed loop experiments. A general framework is given for closed loop identification of coprime factorizations, and it is shown that the freedom that is present in generating appropriate signals for identification can be exploited to obtain (almost) normalized coprime plant factors from closed loop data. A resulting multi-step algorithm is presented and the corresponding asymptotic bias expression is shown to be specifically relevant for evaluating gap-metric distance measures between plant and model. The identification algorithm is illustrated with results that are obtained from closed loop experiments on an open loop unstable mechanical servo system.

Acknowledgement

The authors are grateful to Douwe de Vries and Hans Dötsch for their contributions to the experimental part of the work, and to Gerrit Schootstra and Maarten Steinbuch of the Philips Research Laboratory for their help and support and for access to the CD-player experimental setup.

Appendix

Lemma A.1 [39]. *Consider rational transfer functions* $P_0(z)$ *with right coprime factorization* (N, D) *and* $C(z)$ *with left coprime factorization* $(\tilde{D}_c, \tilde{N}_c)$. *Then* $T(P_0, C) = \begin{bmatrix} P_0 \\ I \end{bmatrix} (I + CP_0)^{-1} \begin{bmatrix} C & I \end{bmatrix}$ *is stable if and only if* $\tilde{D}_c D + \tilde{N}_c N$ *is stable and stably invertible.* □

Proof of Proposition 3.1.
$(a) \Rightarrow (b)$. The mapping $x \rightarrow col(y, u)$ is characterized by the transfer function $\begin{bmatrix} P_0 S_0 F^{-1} \\ S_0 F^{-1} \end{bmatrix}$. By writing $\begin{bmatrix} P_0 S_0 F^{-1} \\ S_0 F^{-1} \end{bmatrix} = \begin{bmatrix} P_0 \\ I \end{bmatrix} (I + CP_0)^{-1} F^{-1}$ and substituting a right coprime factorization (N, D) for P_0, and a left coprime factorization $(\tilde{D}_c, \tilde{N}_c)$ for C we get, after some manipulation, that

$$\begin{bmatrix} P_0 S_0 F^{-1} \\ S_0 F^{-1} \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} (\tilde{D}_c D + \tilde{N}_c N)^{-1} \tilde{D}_c F^{-1}. \tag{A.1}$$

Premultiplication of the latter expression with the stable transfer function $(\tilde{D}_c D + \tilde{N}_c N) \begin{bmatrix} X & Y \end{bmatrix}$ with (X, Y) right Bezout factors of (N, D) shows that $\tilde{D}_c F^{-1}$ is implied to be stable. As a result, $\tilde{D}_c F^{-1} = W$ with W any stable transfer function.
With respect to the mapping $col(y, u) \rightarrow x$, stability of F and FC implies stability of $W^{-1} \begin{bmatrix} \tilde{D}_c & \tilde{N}_c \end{bmatrix}$, which after postmultiplication with the left Bezout factors of $(\tilde{D}_c, \tilde{N}_c)$ implies that W^{-1} is stable.
This proves that $F = W^{-1} \tilde{D}_c$ with W a stable and stably invertible transfer function.
$(b) \Rightarrow (a)$. Stability of F and FC is straightforward. Stability of $S_0 F^{-1}$ and $P_0 S_0 F^{-1}$ follows from (A.1), using the fact that $(\tilde{D}_c D + \tilde{N}_c N)^{-1}$ is stable (lemma A.1). □

References

[1] R.R. Bitmead (1993). Iterative control design approaches. *Proc. 12th IFAC World Congress*, 18-23 July 1993, Sydney, Australia, vol. 9, pp. 381-384.

[2] P.M.M. Bongers and O.H. Bosgra (1990). Low order robust $\mathcal{H}_\infty$ controller synthesis. *Proc. 29th IEEE Conf. Decision and Control*, Honolulu, HI, 194-199.

[3] P.M.M. Bongers and P.S.C. Heuberger (1990). Discrete normalized coprime factorization and fractional balanced reduction. Proc. 9th INRIA Conf. Analysis and Optimiz. Systems. *Lecture Notes in Control and Inform. Sciences, Vol. 144*. Springer Verlag, pp. 307-313.

[4] P.M.M. Bongers and O.H. Bosgra (1990). Low order robust H_∞ controller synthesis. *Proc. 29th IEEE Conf. Decision and Control*, Honolulu, HI, pp. 194-199.

[5] R.A. de Callafon, P.M.J. Van den Hof and M. Steinbuch (1993). Control relevant identification of a compact disc pick-up mechanism. *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, USA.

[6] C.A. Desoer, R.W. Liu, J. Murray and R. Saeks (1980). Feedback system design: the fractional representation approach to analysis and synthesis. *IEEE Trans. Automat. Contr., AC-25*, pp. 399-412.

[7] D.K. de Vries (1994). *Identification of Model Uncertainty for Control Design*. Dr. Dissertation, Delft Univ. Technology, September 1994.

[8] D.K. de Vries and P.M.J. Van den Hof (1995). Quantification of uncertainty in transfer function estimation: a mixed probabilistic - worst-case approach. To appear in *Automatica, Vol. 31*.

[9] T.T. Georgiou and M.C. Smith (1990). Optimal robustness in the gap metric. *IEEE Trans. Automat. Contr., AC-35*, pp. 673-686.

[10] M. Gevers (1993). Towards a joint design of identification and control? In: H.L. Trentelman and J.C. Willems (Eds.), *Essays on Control: Perspectives in the Theory and its Applications*. Birkhäuser, Boston, 1993, pp. 111-151.

[11] G. Gu and P.P. Khargonekar (1992). A class of algorithms for identification in $\mathcal{H}_\infty$. *Automatica, 28*, 299-312.

[12] R.G. Hakvoort, R.J.P. Schrama and P.M.J. Van den Hof (1992). Approximate identification in view of LQG feedback design. *Proc. 1992 Amer. Control Conf.*, pp. 2824-2828.

[13] R.G. Hakvoort (1993b). Worst-case system identification in $\mathcal{H}_\infty$: error bounds and optimal models. *Prepr. 12th IFAC World Congress*, Sydney, vol. 8, pp. 161-164.

[14] R.G. Hakvoort, R.J.P. Schrama, P.M.J. Van den Hof (1994). Approximate identification with closed-loop performance criterion and application to LQG feedback design. *Automatica, vol. 30*, no. 4, pp. 679-690.

[15] R.G. Hakvoort (1994). *Identification for Robust Process Control; nominal models and error bounds*. Dr. Dissertation, Delft Univ. Technology, November 1994.

[16] F.R. Hansen (1989). *Fractional Representation Approach to Closed-Loop System Identification and Experiment Design*. Ph.D.-thesis, Stanford University, Stanford, CA, USA.

[17] A.J. Helmicki, C.A. Jacobson and C.N. Nett (1991). Control-oriented system identification: a worst-case / deterministic approach in $\mathcal{H}_\infty$. *IEEE Trans. Autom. Control, AC-36*, 1163-1176.

[18] P.S.C. Heuberger, P.M.J. Van den Hof and O.H. Bosgra (1992). *A Generalized Orthonormal Basis for Linear Dynamical Systems*. Report N-404, Mech. Engin. Systems and Control Group, Delft Univ. Techn. Short version in CDC'93.

[19] W.S. Lee, B.D.O. Anderson, R.L. Kosut and I.M.Y. Mareels (1992). On adaptive robust control and control-relevant system identification. *Proc. 1992 American Control Conf.*, June 26-28, 1992, Chicago, IL, pp. 2834-2841.

[20] W.S. Lee, B.D.O. Anderson, R.L. Kosut and I.M.Y. Mareels (1993). A new approach to adaptive robust control. *Int. J. Adaptive Control and Signal Proc., vol. 7*, pp. 183-211.

[21] K. Liu and R.E. Skelton (1990). Closed loop identification and iterative controller design. *Proc. 29th IEEE Conf. Decision and Control*, Honolulu, HI, pp. 482-487.

[22] L. Ljung (1987). *System Identification - Theory for the User*. Prentice-Hall, Englewood Cliffs.

[23] D. McFarlane and K. Glover (1988). An H_∞ design procedure using robust stabilization of normalized coprime factors. *Proc. 27th IEEE Conf. Decis. Control*, Austin, TX, pp. 1343-1348.

[24] A.G. Partanen and R.R. Bitmead (1993). Two stage iterative identification/controller design and direct experimental controller refinement. *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 2833-2838.

[25] D.E. Rivera (1991). Control-relevant parameter estimation: a systematic procedure for prefilter design. *Proc. 1991 American Control Conf.*, Boston, MA, pp. 237-241.

[26] R.J.P. Schrama and P.M.M. Bongers (1991). Experimental robustness analysis based on coprime factorizations. In O.H. Bosgra and P.M.J. Van den Hof (Eds.), *Sel. Topics in Identif., Modelling and Contr.*, Vol. 3. Delft Univ. Press, pp. 1-8.

[27] R.J.P. Schrama (1991). An open-loop solution to the approximate closed-loop identification problem. *Proc. 9th IFAC/IFORS Symposium Identification and System Param. Estim.*, July 8-12, 1991, Budapest, Hungary, pp. 1602-1607.

[28] R.J.P. Schrama (1992). *Approximate Identification and Control Design with Application to a Mechanical System*. Dr. Dissertation, Delft University of Technology, May 1992.

[29] R.J.P. Schrama and P.M.J. Van den Hof (1992). An iterative scheme for identification and control design based on coprime factorizations. *Proc. 1992 American Control Conf.*, June 24-26, 1992, Chicago, IL, pp. 2842-2846.

[30] R.J.P. Schrama (1992). Accurate models for control design: the necessity of an iterative scheme. *IEEE Trans. Automat. Contr., AC-37*, pp. 991-994.

[31] R.J.P. Schrama and O.H. Bosgra (1993). Adaptive performance enhancement

by iterative identification and control design. *Int. J. Adaptive Control and Sign. Proc., vol. 7*, no. 5, pp. 475-487.

[32] M. Steinbuch, G. Schootstra and O.H. Bosgra (1992). Robust control of a compact disc player. *Proc. 31st IEEE Conf. Decision and Control*, Tucson, Az, pp. 2596-2600.

[33] M.C. Tsai, E.J.M. Geddes and I. Postlethwaite (1992). Pole-zero cancellations and closed-loop properties of an H∞ - mixed sensitivity design problem. *Automatica, 28*, 519-530.

[34] P.M.J. Van den Hof and R.J.P. Schrama (1993). An indirect method for transfer function estimation from closed loop data. *Automatica, Vol. 28*, No. 6, November 1993.

[35] P.M.J. Van den Hof, R.J.P Schrama, O.H. Bosgra and R.A. de Callafon (1993). Identification of normalized coprime plant factors for iterative model and controller enhancement. *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 2839-2844.

[36] P.M.J. Van den Hof and R.J.P. Schrama (1994). Identification and control - closed loop issues. *Prepr. IFAC Symposium System Identification*, Copenhagen, Denmark, vol. 2, pp. 1-13.

[37] P.M.J. Van den Hof, P.S.C. Heuberger and J. Bokor (1994). Identification with generalized orthonormal basis functions - statistical analysis and error bounds. *Prepr. 10th IFAC Symp. System Identification*, Copenhagen, Denmark, Vol. 3, pp. 207-212.

[38] M. Vidyasagar (1984). The graph metric for unstable plants and robustness estimates for feedback stability. *IEEE Trans. Automat. Contr., AC-29*, pp. 403-417.

[39] M. Vidyasagar (1985). *Control Systems Synthesis: A Factorization Approach*. M.I.T. Press, Cambridge, MA, USA.

[40] M. Vidyasagar (1988). Normalized coprime factorizations for nonstrictly proper systems. *IEEE Trans. Automat. Contr., AC-33*, pp. 300-301.

[41] Z. Zang, R.R. Bitmead and M. Gevers (1991). *Iterative Model Refinement and Control Robustness Enhancement*. Report 91.137, CESAME, Univ. Louvain-la-Neuve, November 1991. See also *Proc. 30th IEEE Conf. Decision and Control*, Brighton, UK, pp. 279-284.

[42] Y.C. Zhu and A.A. Stoorvogel (1992). Closed loop identification of coprime factors. *Proc. 31st IEEE Conf. Decision and Control*, Tucson, AZ, pp. 453-454.

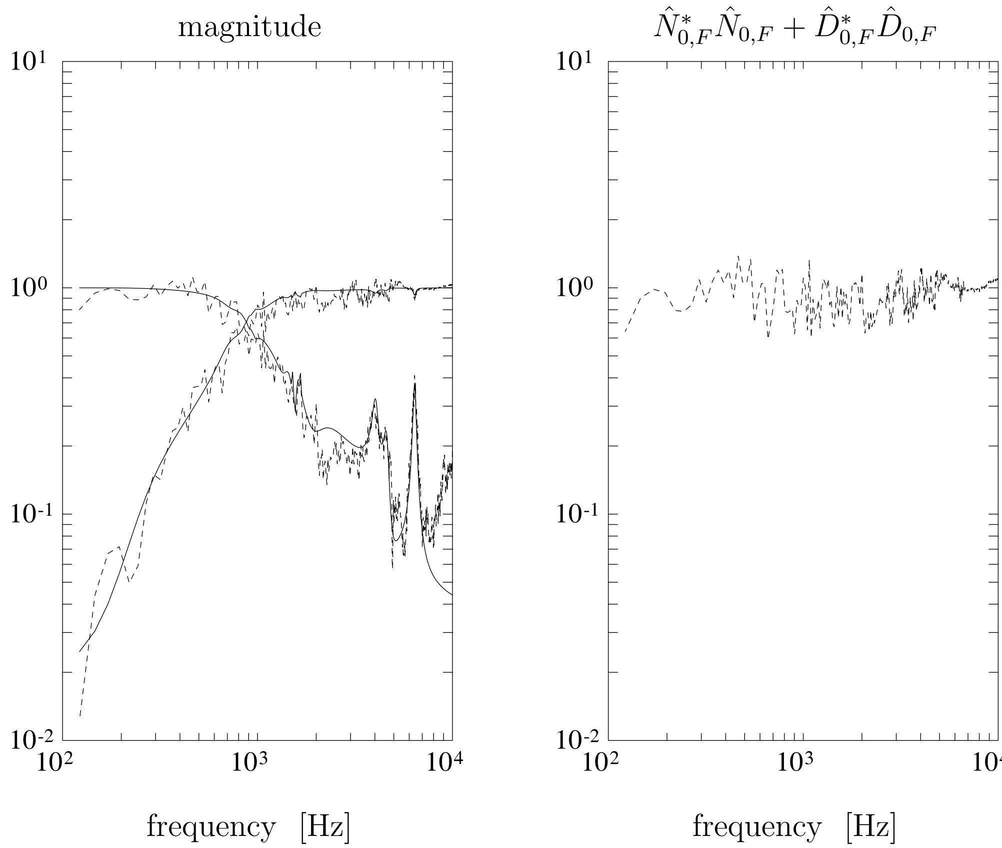

Fig. 4: Bode magnitude plots of the results in Step 1 of the procedure:

a: Identified 32nd order coprime plant factors (N_x, D_x) of auxiliary model P_x (solid) and spectral estimates $(\hat{N}_{0,F}, \hat{D}_{0,F})$ of the factorization $(N_{0,F}, D_{0,F})$ (dashed).

b: Plot of $\hat{N}^*_{0,F}\hat{N}_{0,F} + \hat{D}^*_{0,F}\hat{D}_{0,F}$ using the spectral estimates $\hat{N}_{0,F}$ and $\hat{D}_{0,F}$.

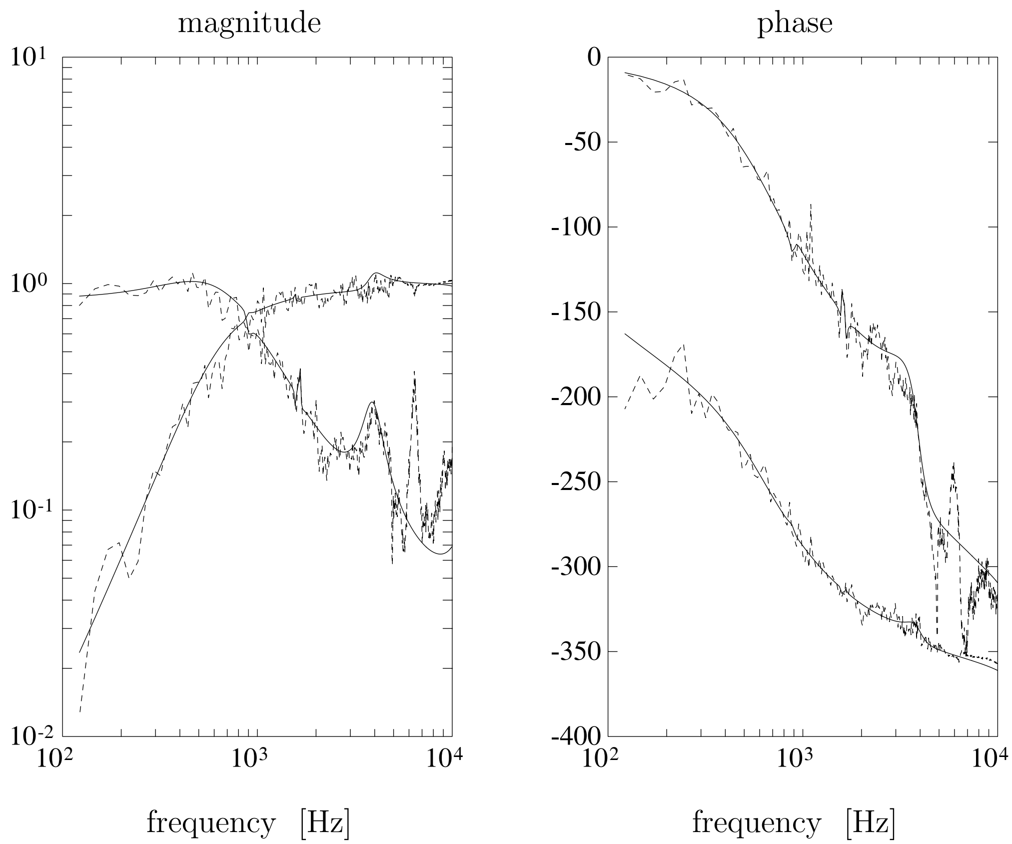

Fig. 5: Bode plot of the results in Step 2 of the procedure: Identified 10th order coprime factors $(\hat{N}, \hat{D})$ (solid) and spectral estimates $(\hat{N}_{0,F}, \hat{D}_{0,F})$ (dashed) of the factorization $(N_{0,F}, D_{0,F})$.

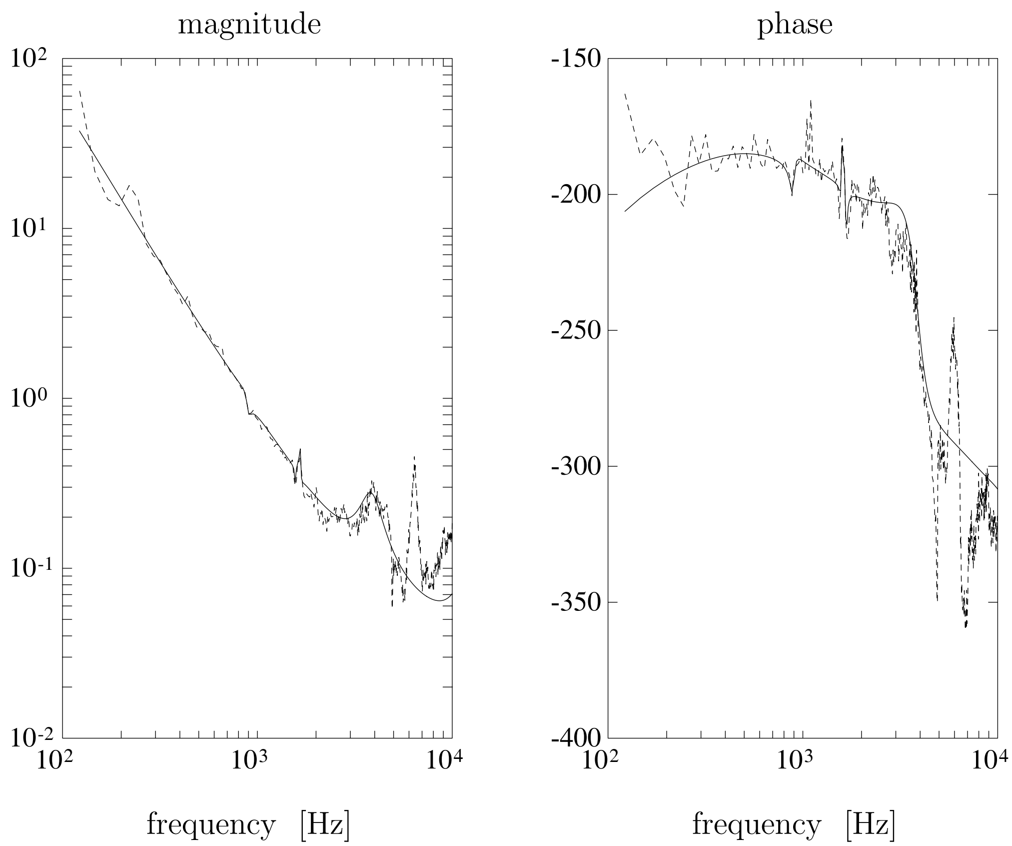

Fig. 6: Bode plot of the results in Step 2 of the procedure: Identified 10th order model $\hat{P} = \hat{N}\hat{D}^{-1}$ (solid) and spectral estimate $\hat{N}_{0,F}\hat{D}_{0,F}^{-1}$ (dashed) of the plant P_0.