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ADAPTIVE PERIODIC NOISE CANCELATION FOR COOLING FANS WITH VARYING ROTATIONAL SPEED

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ABSTRACT

This paper presents a novel method of simultaneously tracking and rejecting time-varying sinusoids in the presence of random noise by using feedback control. The technique extends the internal model-principle by using an extended Kalman filter to create time-varying gains and a time-varying internal model. The state feedback gain, however, is not time-varying and is designed using standard time-invariant LQR methods. This control algorithm is applied to active noise cancelation and in simulations is shown to converge quickly in the presence of noise. Methods of improving convergence of this algorithm are discussed.

NOMENCLATURE

A You may include nomenclature here.
α There are two arguments for each entry of the nomemclature environment, the symbol and the definition.
Feedback Microphone Mic. used to measure and feedback noise.
Noise canceling speaker
Acoustic Noise
Error path Feedback path

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Introduction

Many time-invariant systems are subjected to time-varying disturbances. Moreover, the future values of the disturbances are not typically known and therefore standard time-varying optimal control cannot be applied and may not be the most appropriate since the system itself is not time-varying. Examples of these types of systems are acoustical, vibrational, and repetitive systems such as cooling systems with periodic fan noise, structures subjected to earthquakes, and hard disk drives.

These class of systems have been studied and for a fixed frequency it has been shown that an internal model based controller, controller based upon the internal model principle [1], will completely reject the periodic disturbances even under parametric uncertainty [2]. In this paper, we address the problem of extending this work for rejecting a disturbance with a fixed and known frequency to an unknown and possible time-varying frequency. To study the proposed algorithm an acoustical system subjected to a time-varying periodic disturbance is simulated.

Active noise cancelation [3] typically uses feedforward based adaptive algorithms like the least-mean-square (LMS), filtered-X LMS (FXLMS), adaptive notch filters, recursive least squares (RLS), and variations of the aforementioned. In the case

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of acoustic feedback these method need modification to work properly [4]. Designing feedback controllers based upon the internal model principle has produced good results in the area of ANC, and the authors in [5] reported a $30-40$ dB reduction in an acoustic duct. The internal model principle and convex optimization was used to find a controller that rejects periodic disturbances in [6]. The method contained therein requires that the plant has no zeros in common with the poles of the disturbance and optimizes the closed loop system over the zeros of the controller. In [7] a feedback controller was designed and implemented that successfully eliminated the first four harmonic frequencies of the disturbance. In this paper, we use an feedback controller that adapts to the time-varying disturbances for active noise cancelation.

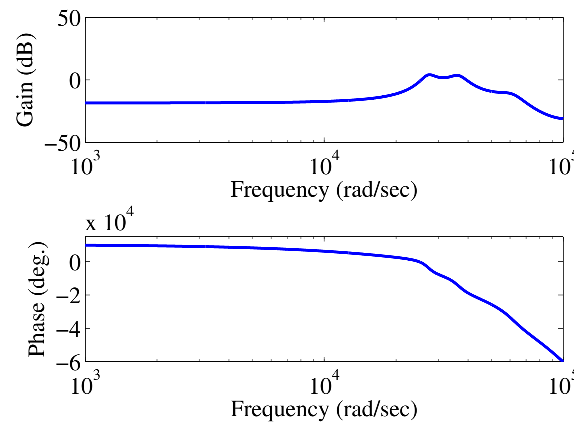

Figure 2. Frequency response of acoustic system.

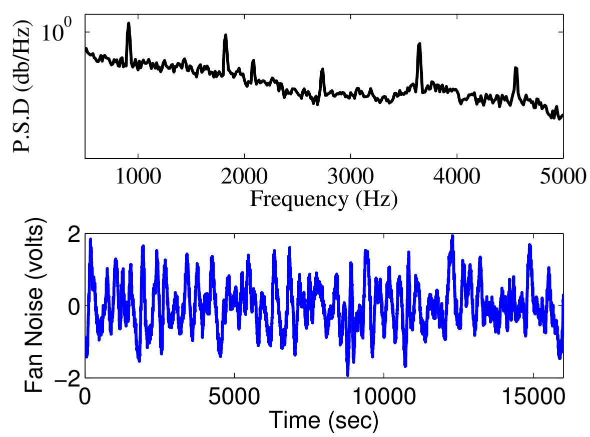

Figure 3. Typical acoustic noise from a cooling fan.

Fan and Enclosure Acoustics

Description

The system we are considering is depicted in Fig. 1. The acoustic noise from a cooling fan propagates down a cylindrical duct with a feedback active noise cancelation system mounted at the end. The feedback microphones are used to measure the acoustic noise and the noise canceling speakers are used to create anti-noise that will cancel the undesired acoustic noise from the fan. The microphones are mounted inside acoustical foam to help shield them from windage that causes a significant amount of measurement errors. All of the speakers are actuated with the same signal, so as to act like one larger speaker. The same for the microphones, several are mounted around to opening and the average of the signal is used for the feedback control. The averaging process reduces the measurement noise.

The frequency response of the plant is shown in Fig. 2. The input to the system is the signal sent to the noise canceling speaker and the output is the signal measured by the feedback microphone and therefore the system is dynamics of the speaker, microphone, and the acoustics. The system was estimated using standard system identification [8] techniques.

The acoustic noise of a typical cooling fan is shown in Fig. 3. On the top the power spectral density PSD is shown and on the bottom the a time series of a microphone signal is shown. From both figures it should be clear that the fan noise contains some broadband noise and harmonic noise. The harmonic noise, or sinusoids with frequencies that are multiples of each other, is due to the blade pass frequency of the cooling fan [9].

Model of System

The dynamics of the acoustic system or *plant* will be modeled with the state space model

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B_u u(k) + B_w w(k) \\ y_p(k) &= C_p x_p(k) + D_{yw} w(k) + d(k) \\ z(k) &= C_z x_p(K) + D_{zu} u(k) \end{aligned}, \tag{1}$$

where $x_p(k) \in \mathbb{R}^{n_p}$ are the plant states, $u(k) \in \mathbb{R}^{n_u}$ is the control signal, $w(k) \in \mathbb{R}^{n_w}$ is white noise, $d(k) \in \mathbb{R}^{n_y}$ is the time-varying periodic disturbance, $y_p(k) \in \mathbb{R}^{n_y}$ is the measurable output of the plant, and $z(k) \in \mathbb{R}^{n_z}$ is the performance channel. It will also be assumed throughout the paper that $D_{yw}D_{yw}^T > 0$, $B_w D_{yw} = 0$, and $D_{zu}^T D_{zu} > 0$. Typically, the performance channel is a weighted version of the output to reflect the various performance measures

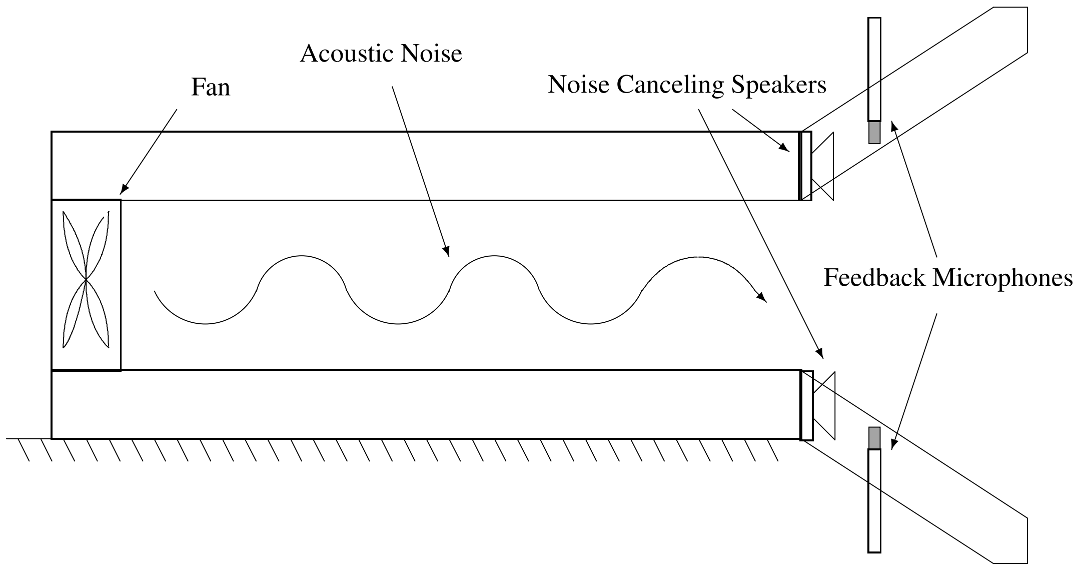

Figure 1. Active noise canceling system used to eliminate the unwanted noise from a cooling fan.

that one desires to reduce.

For the purposes of the control design, a model of the disturbance or *internal model* will be used and is given by

$$\begin{aligned} x_m(k+1) &= \mathcal{A}_m(x_m(k))x_m(k) + B_m u_m(k) \\ y_m(k) &= C_m x_m(k) \end{aligned},$$

where $x_m(k) \in \mathbb{R}^{n_m}$ are the internal model states, $u_m(k) \in \mathbb{R}^{n_y}$ is the input to the internal model, and $y_m(k) \in \mathbb{R}^{n_{ym}}$ is the output of the internal model.

The overall goal is to eliminate unwanted noise that appears as a time-varying periodic signal and random noise due to measurement errors and broadband noise. This can be achieved by eliminating the periodic components and further reducing the mean square pressure by using an LQG style of controller.

Controller Structure

It was shown in [10] that the LTI controller given by

$$C(q) = \left[\begin{array}{cc|c} A_p - L_pC_p - B_uK & 0 & L_p \\ L_mC_p & A_m & -L_m \\ \hline -K & C_m & 0 \end{array}\right], \qquad (2)$$

is an internal model-based controllers and has an interpretation of a learning controller when the appropriate internal model is chosen. In this controller A_m and C_m are given matrices that serve as a model of the disturbance called the internal model and the gains L_p, L_m, and K are designed to maintain stability and performance. This controller is able to cancel periodic disturbances that have a fixed, known frequency.

In this paper, we use this LTI controller as a starting point to design an adaptive internal model-based controller that can track and cancel time-varying periodic disturbances and simultaneously cope with the random disturbances in the system. To accomplish this task, the internal model is parameterized as a function of ω. This implies that the observer gains in Eq. (2) necessarily will change as a function of ω.

Following the same methodology as [11] the controller is parameterized with respect to ω this gives

$$C(q,\omega) = \left[\begin{array}{cc|c} A_p - L_p(\omega)C_p - B_uK(\omega) & 0 & L_p(\omega) \\ L_m(\omega)C_p & A_m(\omega) & -L_m(\omega) \\ \hline -K(\omega) & C_m & 0 \end{array}\right], \qquad (3)$$

where the controller gains must be designed so that the closed loop system is stable and has desirable properties.

With the controller given by Eq. (3) and the system given by

Eq. (1) the closed loop system becomes

$$\begin{bmatrix} x_p(k+1) \\ x_c^{(1)}(k+1) \\ x_c^{(2)}(k+1) \end{bmatrix} =$$

$$\begin{bmatrix} A_p & -B_uK(\omega) & B_uC_m \\ L_p(\omega)C_p & A_p - L_p(\omega)C_p - B_uK(\omega) & 0 \\ -L_m(\omega)C_p & L_m(\omega)C_p & A_m(\omega) \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_c^{(1)}(k) \\ x_c^{(2)}(k) \end{bmatrix}$$

$$+ \begin{bmatrix} B_w \\ L_p(\omega)D_{yw} \\ -L_m(\omega)D_{yw} \end{bmatrix} w(k) + \begin{bmatrix} 0 \\ L_p(\omega) \\ -L_m(\omega) \end{bmatrix} d(k)$$

$$z(k) = \begin{bmatrix} C_z & -D_{zu}K(\omega) & D_{zu}C_m \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_c^{(1)}(k) \\ x_c^{(2)}(k) \end{bmatrix},$$

where the controller states have been partitioned into $x_c^{(1)}(k)$ and $x_c^{(2)}(k)$. Applying the similarity transformation $T = \begin{bmatrix} I & -I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$ and rearranging the states gives the following equivalent realization

$$\begin{bmatrix} \bar{x}_p(k+1) \\ \bar{x}_c^{(2)}(k+1) \\ \bar{x}_c^{(1)}(k+1) \end{bmatrix} =$$

$$\begin{bmatrix} A_p - L_p(\omega)C_p & B_uC_m & 0 \\ -L_m(\omega)C_p & A_m(\omega) & 0 \\ L_p(\omega)C_m & 0 & A_p - B_uK(\omega) \end{bmatrix} \begin{bmatrix} \bar{x}_p(k) \\ \bar{x}_c^{(2)}(k) \\ \bar{x}_c^{(1)}(k) \end{bmatrix}$$

$$+ \begin{bmatrix} B_w - L_p(\omega)D_{yw} \\ -L_m(\omega)D_{yw} \\ L_p(\omega)D_{yw} \end{bmatrix} w(k) + \begin{bmatrix} -L_p(\omega) \\ -L_m(\omega) \\ L_p(\omega) \end{bmatrix} d(k)$$

$$z(k) = \begin{bmatrix} C_z & D_{zu}C_m & C_z - D_{zu}K(\omega) \end{bmatrix} \begin{bmatrix} \bar{x}_p(k) \\ \bar{x}_c^{(2)}(k) \\ \bar{x}_c^{(1)}(k) \end{bmatrix}$$

where is it obvious that the design process becomes the design of a state feedback gain for Eq. (1) and an observer gain for the series connection of the internal model and plant. The design of the state feedback gain is straightforward with the use of LQR theory and results in a discrete time Riccati equation. The observer gain is slightly more complicated and an Extended Kalman Predictor is needed since the frequency of the disturbance is also unknown. The following section describes how to find the gains for the controller given in Eq. (3).

Adaptive Controller Synthesis

Internal Model

In order for the frequency estimation and cancelation to work properly it is important to construct an internal model that captures the relevant properties of the unknown signal. In our case, a model of a periodic signal with a time-varying frequency is needed.

It was shown in [12] that a continuous time model that represents the periodic signal $y(t) = \cos(\omega t + \phi)$ with a time-varying frequency is given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_d(t)^2 & \dot{\omega}_d(t)\frac{1}{\omega_d(t)} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$
$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

where $\omega_d(t) = \frac{d}{dt}(\omega t + \phi)$ and another realization is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} 0 & \omega_d(t) \\ -\omega_d(t) & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned} \tag{4}$$

Notice that one must be careful when choosing the realization, since the normal LTI realizations are not always the same as the time-varying realizations.

Applying a zero-order hold, with sample time Δt, to Eq. (4) gives

$$\begin{aligned} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} &= \begin{bmatrix} \cos(\omega\Delta t) & \sin(\omega\Delta t) \\ -\sin(\omega\Delta t) & \cos(\omega\Delta t) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ y(k) &= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \end{aligned} \tag{5}$$

where $\omega := \omega_d(t)|_{t=k}$ is held constant over the samping interval. This model will be used as the building blocks of an internal model. The part that remains is chose how the input should affect the states of the internal model. This choice is important since the internal model will be inside the feedback loop and thus will change the dynamics of the closed loop system.

Since the realization Eq. (5) does not have any zeros (it doesn't have an input), we will choose an internal model that has an input but does not have any zeros. One such realization is given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} \cos(\omega\Delta t) & \sin(\omega\Delta t) \\ -\sin(\omega\Delta t) & \cos(\omega\Delta t) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}.$$

Since we require that the controller estimate the frequency of the disturbance as well as reject it, ω will also be estimated. This means that ω is a state and should be added to the internal model, this gives

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \omega(k+1) \end{bmatrix} = \begin{bmatrix} \cos(\omega(k)\Delta t) & \sin(\omega(k)\Delta t) & 0 \\ -\sin(\omega(k)\Delta t) & \cos(\omega(k)\Delta t) & 0 \\ 0 & 0 & 1-\varepsilon \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \omega(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \omega(k) \end{bmatrix}$$

where it was assumed that the frequency of the disturbance is slowly varying. Notice that this is a nonlinear system since $\omega(k)$ appears inside the $\cos(\cdot)$ and $\sin(\cdot)$ functions. For this reason an Extended Kalman Predictor will be used. This realization has also been used previously (see [13], for example) to track frequencies and will be used for the remainder of the paper as a subsystem for the internal model. The subsystems need to be connected to accommodate disturbances that may contain more than one sinusoid.

To build an internal model that can accommodate any disturbance signal that is a combination of N distinct sinusoids, define

$$A(\omega_i) = \begin{bmatrix} \cos(\omega_i(k)\Delta t) & \sin(\omega_i(k)\Delta t) & 0 \\ -\sin(\omega_i(k)\Delta t) & \cos(\omega_i(k)\Delta t) & 0 \\ 0 & 0 & 1-\varepsilon \end{bmatrix} \quad B(\omega_i) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C(\omega_i) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

then connect the subsystems in series. The result is

$$A_m = \begin{bmatrix} A(\omega_1) & B(\omega_1)C(\omega_2) & 0 & \dots & 0 \\ 0 & A(\omega_2) & B(\omega_2)C(\omega_3) & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \ddots & A(\omega_N) \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ B(\omega_N) \end{bmatrix} \qquad C_m = \begin{bmatrix} C(\omega_1) & 0 & \dots & 0 \end{bmatrix}$$

for the state space matrices that define the internal model. Or, connect them in parallel to get

$$A_m = \begin{bmatrix} A(\omega_1) & 0 & 0 & \dots & 0 \\ 0 & A(\omega_2) & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \ddots & A(\omega_N) \end{bmatrix}$$

$$B_m = \begin{bmatrix} B(\omega_1) \\ B(\omega_2) \\ \vdots \\ B(\omega_N) \end{bmatrix} \qquad C_m = \begin{bmatrix} C(\omega_1) & C(\omega_2) & \dots & C(\omega_N) \end{bmatrix}.$$

There are many options for the connection of the subsystems as well as internal model and the user must choose the appropriate method for the application.

State Feedback Gain

In this section the state feedback gain is found using some results from LQR theory. The goal is to find a state feedback gain that stabilizes the plant Eq. (1) while minimizing the energy of the output channel $z(k)$. To design the gain K consider the following optimization problem:

$$\min_{u(0),u(1),\dots} \sum_{k=0}^{\infty} x_p(k)^T C_z^T C_z x_p(k) + u(k)^T D_{zu}^T D_{zu} u(k) \qquad \text{s.t.} \quad (6)$$

$$x_p(k+1) = A_p x_p(k) + B_u u(k) \qquad (7)$$

If (A_p, B_u) is controllable, (A_p, C_z) is observable, $C_z^T C_z \geq 0$, and $D_{zu}^T D_{zu} > 0$ then the optimal solution is given by

$$u = K x_p(k)$$

$$K^* = (B_u^T P_c B_u + D_{zu}^T D_{zu})^{-1} B_u^T P_c A_p$$

where P_c satisfies

$$P_c = A_p^T P_c A_P - A_p^T P_c B_u (B_u^T P_c B_u + D_{zu}^T D_{zu})^{-1} B_u^T P_c A_p + C_z^T C_z. \tag{8}$$

Designing the state feedback gain in this manner acheives two goals. First it guarantees that $A_P - B_u K$ is stable which is important since the eigenvalues of the closed loop system contain the eigenvalues of $A_p - B_u K$. Secondly, if the observer error is zero then the energy of the output is minimized. Of course, the observer error will not be zero, but if it is small then the controller is expected to perform well. The design of the observer becomes crucial for the performance and stability of the system and is discussed in the following sections.

Extended Kalman Filter

Before applying the Extended Kalman Filter (EKF) [14] to the control scheme we are considering, a short review of this nonlinear filtering technique is given. Consider the nonlinear discrete time system given by

$$x(k+1) = f(x(k), u(k), w(k)) \tag{9}$$
$$y(k) = h(x(k), u(k), v(k)), \tag{10}$$

where $x(k)$ are the states, $u(k)$ are deterministic inputs, and $w(k)$ and $v(k)$ are zero mean, gaussian white noise sequences satisfying

$$\mathbb{E}\{w(k)w(k)^T\} = Q(k) \tag{11}$$
$$\mathbb{E}\{v(k)v(k)^T\} = R(k). \tag{12}$$

The EKF is given by the following:

Algorithm 1 (Extended Kalman Filter).

Initialize:

$$P(0|0) = cov(x(0)) \quad \hat{x}(0|0) = \mathbb{E}\{x(0)\}$$

For k=0,1,2,...

Predict:

$$\hat{x}(k+1|k) = f(\hat{x}(k|k), u(k), 0)$$
$$P(k+1|k) = F(k)P(k|k)F(k)^T + Q(k)$$

where

$$F(k) := \frac{\partial}{\partial x} f(x,u,w)\Big|_{\substack{x=\hat{x}(k|k)\\ u=u(k)\\ w=0}}$$

Update:

$$S(k+1) = (H(k+1)P(k+1|k)H(k+1)^T + R(k))$$
$$K(k+1) = P(k+1|k)H(k+1)^T S(k+1)^{-1}$$
$$P(k+1|k+1) = (I - K(k+1)H(k+1))P(k+1|k)$$
$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)(y(k+1) - h(\hat{x}(k+1|k), 0))$$

where

$$H(k+1) := \frac{\partial}{\partial x} h(x,u,v)\Big|_{\substack{x=\hat{x}(k+1|k)\\ u=u(k)\\ w=0}}$$

There are several variations of this algorithm, but the above version will be used for this paper.

Extended Kalman Predictor for State and Frequency Estimation

In this section we apply the EKF to our control problem to create a nonlinear estimator that tracks the states and the periodic disturbances in the presence of random noise. Since we want to implement our algorithm in discrete time it is required that the control signal is a function of past outputs and therefore we arrive at a predictor. Recall, that the control design is composed of two sub-problems: an observer design and a state feedback design. To embed the internal model into the controller the observer design is done for the series connection of the internal model and plant. Additionally, the internal model states contain an estimate of the disturbance frequency so we will be able to estimate the frequency and states simultaneously.

To design the estimator, we will assume that the input into the internal model is white noise by setting $u_m(k) = w(k)$. The series connection of the nonlinear internal model and the linear time invariant plant is given by

$$\begin{bmatrix} x_p(k+1) \\ x_m(k+1) \end{bmatrix} = \begin{bmatrix} A_p & B_u C_m \\ 0 & A_m(x_m(k)) \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_m(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_m \end{bmatrix} u_m(k) + \begin{bmatrix} B_w \\ 0 \end{bmatrix} w(k) \tag{13}$$
$$y_p(k) = \begin{bmatrix} C_p & 0 \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_m(k) \end{bmatrix} + D_{yw} w(k)$$

which can be expressed as

$$\begin{aligned} x(k+1) &= \mathcal{A}(x(k))x(k) + \mathcal{B}_w w(k) \\ y_p(k) &= \mathcal{C}_p x(k) + D_{yw} w(k) \end{aligned} \tag{14}$$

or as

$$\begin{aligned} x(k+1) &= f(x(k), w(k)) \\ y_p(k) &= h(x(k), w(k)). \end{aligned} \tag{15}$$

In the form given in Eq. (14) we will apply the EKF to find the predictor gains. For notational convenience define

$$\begin{aligned} Q(k) &:= \mathcal{B}_w \mathcal{B}_w^T \\ R(k) &:= D_{yw} D_{yw}^T \end{aligned}$$

and assume $\mathcal{B}_w D_{yw}^T = 0$. If we apply Algorithm 1 to the system in Eq. (14) we arrive at the following:

Initialize:

$$P(0|0) = cov(x(0)) \quad \hat{x}(0|0) = \mathbb{E}\{x(0)\}$$

For k=0,1,2,...

Predict:

$$\begin{aligned} \hat{x}(k+1|k) &= \mathcal{A}(\hat{x}(k|k))\hat{x}(k|k) \\ &= (\mathcal{A}(\hat{x}(k|k)) - \mathcal{L}(k)\mathcal{C}_p)\hat{x}(k|k-1) + \mathcal{L}(k)y(k) \\ P(k+1|k) &= F(k)P(k|k)F(k)^T + Q(k) \end{aligned}$$

where

$$\mathcal{L}(k) = \mathcal{A}(\hat{x}(k|k))K(k)$$
$$F(k) := \frac{\partial}{\partial x}(\mathcal{A}(x)x)\bigg|_{x = \hat{x}(k|k)}$$

Update:

$$\begin{aligned} K(k+1) &= P(k+1|k)\mathcal{C}_p^T(\mathcal{C}_p P(k+1|k)\mathcal{C}_p^T + R(k))^{-1} \\ \hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + K(k+1)(y(k+1) - \mathcal{C}_p\hat{x}(k+1|k)) \\ P(k+1|k+1) &= (I - K(k+1)\mathcal{C}_p)P(k+1|k) \end{aligned}$$

Notice that the filtering gain $\mathcal{L}(k)$ is a function of $y(k)$, but for implementation it is required that the gains be a function of past outputs. Therefore we will replace $\hat{x}(k|k)$ with $\hat{x}(k|k-1)$ so that the algorithm is easy to implement and denote this algorithm the Extended Kalman Predictor EKP. Additionally, if the update and prediction error covariances are combined the simplified EKP Algorithm for Frequency Tracking is obtained and shown on the right.

This algorithm is used to update the values of the controller. Specifically the gains

$$\begin{bmatrix} L_p(k) \\ -L_m(k) \end{bmatrix} = \mathcal{L}(k),$$

and the internal model $A_m(k) = A_m(\hat{x}(k|k-1))$ are updated to stabilize the system and simultaneously track the unknown and possibly time-varying frequency of the disturbance.

Convergence

For a linear system $P(k|k-1)$ is defined as the covariance of the error given all of the previous information or

$$\begin{aligned} P(k|k-1) &= \mathbb{E}\{(\hat{x}(k) - x(k))(\hat{x}(k) - x(k))^T | Y_{k-1}\} \\ Y_{k-1} &= \{u_m(0), y_p(0), u_m(1), y_p(1), \ldots, u_m(k-1), y_p(k-1)\} \end{aligned}$$

and $Q(k)$ has the interpretation of the amount of noise or uncertainty on the states and is given by $\mathbb{E}\{w(k)w(k)^T\}$. If $Q(k)$ is large (meaning in comparison to $R(k)$) then the states will change more rapidly. If $Q(k)$ is small then more averaging will occur to smooth out the measurement noise described by $R(k)$. Therefore if the estimated frequency ω is far from the true frequency ω_0 then setting the appropriate entry of $Q(k)$, the entry that would

Algorithm 2 (EKP for Frequency Tracking).

Initialize:

$$P(1|0) = F(0)cov(x(0))F(0) + Q(0) \quad \hat{x}(0|0) = \mathbb{E}\{x(0)\}$$
$$\hat{x}(1|0) = f(\hat{x}(0|0), 0, 0)$$

where

$$F(0) := \frac{\partial}{\partial x}(\mathcal{A}(x)x)\bigg|_{x=\hat{x}(0|0)}$$

For k=1,2,...

Predict:

$$\hat{x}(k+1|k) = (\mathcal{A}(\hat{x}(k|k-1)) - \mathcal{L}(k)\mathcal{C}_p)\hat{x}(k|k-1) + \mathcal{L}(k)y(k)$$
$$P(k+1|k) = F(k)P(k|k-1)F(k)^T - F(k)P(k|k-1)\mathcal{C}p^T(\mathcal{C}_p)^{-1}\mathcal{C}pP(k|k-1)F(k)^T + Q(k)$$

where

$$\mathcal{L}(k) = \mathcal{A}(\hat{x}(k|k-1))$$
$$K(k) = P(k|k-1)\mathcal{C}_p^T(\mathcal{C}_pP(k|k-1)\mathcal{C}_p^T + R(k))^{-1}$$
$$F(k) := \frac{\partial}{\partial x}(\mathcal{A}(x)x)\bigg|_{x=\hat{x}(k|k-1)}$$

represent the error on ω, to be large will cause faster convergence. When ω is near ω_0 then the entry of $Q(k)$ should be small. Furthermore it was noted in [13] that the error on the remaining internal model states should be small since they are not unknown.

This can be accomplished by setting

$$Q(k) = A_1(1-\varepsilon)^(k-m) + A_2$$

with A_1 large,A_2 small, and $m = 0$ at intialization. If the error between the estimated and true frequency grows then a trigger can be used to reset $Q(k)$. A potential trigger is

$$\text{if} \quad \sum_{l=0}^{n} |y(k-l)| > \beta \qquad \text{then } m = k. \tag{16}$$

Simulation

The disturbance that is used in this simulation is given by

$$d(t) = A_1\sin(\alpha(t)) + A_2\sin(2*\alpha(t)),$$

where the frequency is $\omega_0(t) = \frac{d}{dt}\alpha(t)$. The internal model is

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \\ \omega(k+1) \end{bmatrix} = \begin{bmatrix} \cos(\omega(k)\Delta t) & \sin(\omega(k)\Delta t) & 0 & 0 & 0 \\ -\sin(\omega(k)\Delta t) & \cos(\omega(k)\Delta t) & 0 & 0 & 0 \\ 0 & 0 & \cos(2\omega(k)\Delta t) & \sin(2\omega(k)\Delta t) & 0 \\ 0 & 0 & -\sin(2\omega(k)\Delta t) & \cos(2\omega(k)\Delta t) & 0 \\ 0 & 0 & 0 & 0 & 1-\varepsilon \end{bmatrix} \times \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ \omega(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \\ \omega(k) \end{bmatrix}$$

where the initial estimate of the frequency ω is set to 1000 rad/sec and $Q(k)$ is given by

$$Q(k) = A_1(1-\varepsilon)^(k-m) + A_2 \tag{17}$$

$$A_1 = \begin{bmatrix} I & 0 \\ 0 & 1000 \end{bmatrix} \qquad A_2 = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix} \tag{18}$$

where m is determined by the trigger given in Eq. (16).

In order to model two sinusoids, 4 states are required. In addition the frequency of the sinusoids needs to be estimated and in this case that means an additional state. In general there would be an additional state for each sinusoid, but since this method allows the user to parameterize the internal model to match the disturbance only one frequency needs to be estimated when tracking a base frequency and its harmonics.

In Fig. 5 the convergence of the estimated frequency to the true frequency is shown. The dashed line is the true frequency and the solid line is the estimated frequency ω. This algorithm is able to converge to the true frequency very quickly. The results of applying the controller Eq. (2) to the plant is shown in Fig. 4. In this figure, the output of the plant before and after control are shown, dotted and solid respectively. Notice that the only the random noise is left in the output of the plant after the control is applied. From both of these figures it can be seen that the algorithm has difficulty tracking the ramp part of the true frequency. This is due to the dynamics of the estimator and could be changed if desired. However, the estimator tracks constant frequencies very well as designed.

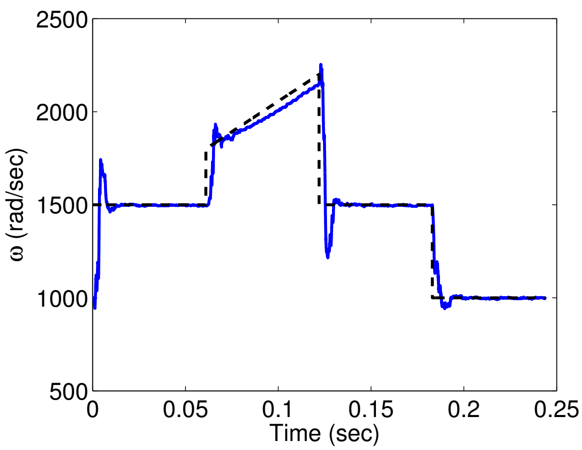

Figure 5. Convergence of estimated frequency. The true frequency is dashed and the estimate is solid.

If the entry of $Q(k)$, the last column and row,. that represents the error on ω is denoted by $q(k)$ then the size of this uncertainty as a function of time is shown in Fig. 6. The sudden jumps are due to the trigger. The estimate error grows, then the controller doesn't suppress the sinusoids in the output, this triggers the $q(k)$ and the estimator converges again. Notice that the jumps in $q(k)$ correspond to quick changes in frequency as seen in Fig. 5.

Conclusions

In this paper a new algorithm is presented that uses an Extended Kalman Predictor in combination with an internal model-based controller to simultaneously track and reject a time-varying periodic disturbance. The control algorithm was applied to active noise control where the periodic disturbance is a sum of sinusoids whose frequencies are multiples of each other. The design of the controller is achieved in two steps by relying upon the well-known separation principle. One step is a state feedback design where the state feedback gain is not time-varying. The other step is the Extended Kalman Predictor and the resulting predictor gain and internal model are time-varying so that the frequency of the disturbance is rejected.

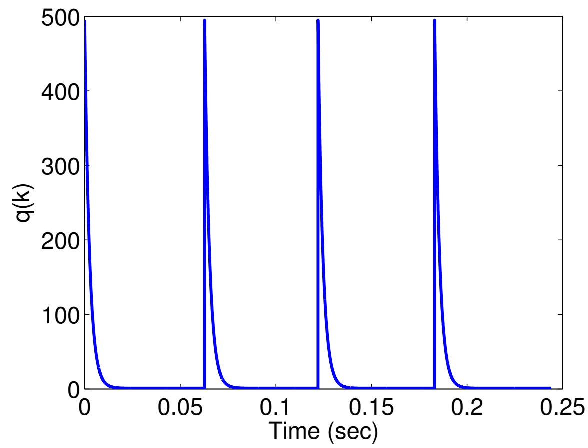

Figure 6. Size of convergence parameter q(k).

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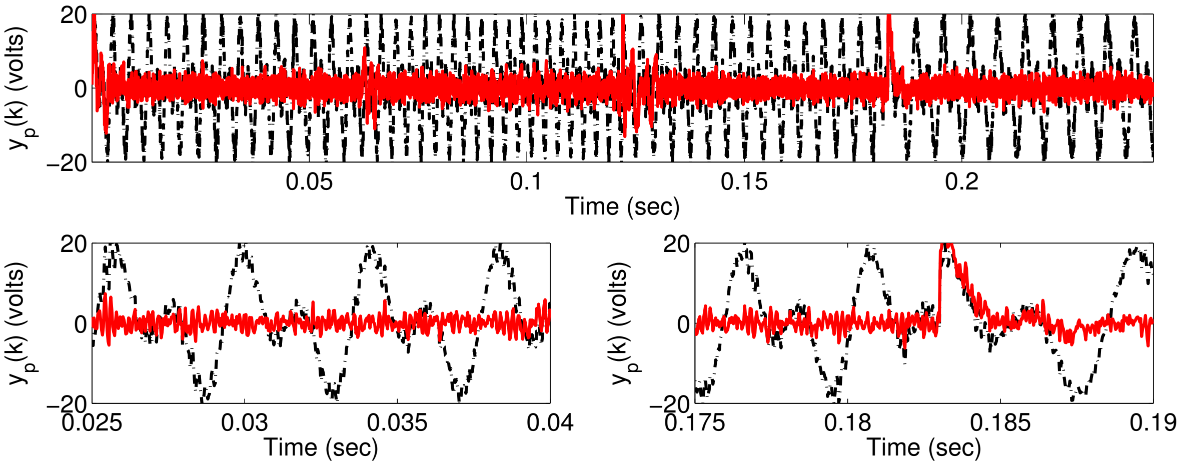

Figure 4. The output of the plant before and after control, dotted and solid respectively.

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