MAE 110A - Sample Homework Solution

1.1 PROBLEM DESCRIPTION:

A FRICTIONLESS PISTON IS RAISED SLOWLY BY HEATING THE GAS CONTAINED IN THE CYLINDER.

GIVEN:

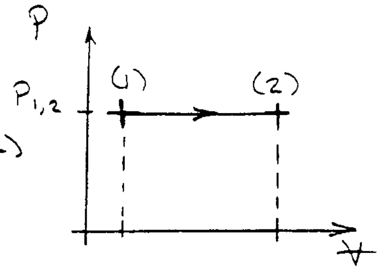
$p_1 = 0.2$ MPa

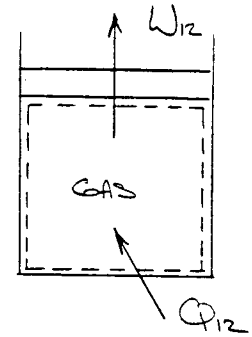
$p_2 = p_1$ (CONSTANT PRESSURE)

$V_1 = 1.0\ m^3$

$V_2 = 2.0\ m^3$

$Q_{12} = 2000$ kJ $= 2$ MJ





DETERMINE: CHANGE IN INTERNAL ENERGY, ΔU_{12}

ENGINEERING MODEL:
1. QUASIEQUILIBRIUM PROCESS
2. SYSTEM IS THE GAS ONLY.
3. NEGLIGIBLE KINETIC AND POTENTIAL ENERGY EFFECTS.

BASIC EQUATIONS:

1st LAW FOR CLOSED SYSTEM: $\Delta KE + \Delta PE + \Delta U = Q_{IN} + W_{IN} - Q_{out} - W_{out}$ (1)

WORK: $W_{12} = \int_1^2 p\,dV$ (2)

STEPS:

APPLY 1ST LAW TO SYSTEM:

$$Q_{IN_{12}} = W_{OUT_{12}} + \cancel{\Delta KE}^{③} + \cancel{\Delta PE}^{③} + \Delta U_{12}$$

SOLVING FOR ΔU_{12}: $\Delta U_{12} = Q_{IN,12} - W_{OUT,12}$ (3)

TO DETERMINE $W_{OUT,12}$, USE EQU. (2):

$$W_{OUT,12} = |W_{12}| = \left| \int_1^2 p\,dV \right|$$

$$W_{OUT,12} = |p(V_2 - V_1)| \quad (4)$$

SUBSTITUTING (4) INTO (3):

$$\Rightarrow \Delta U_{12} = U_2 - U_1 = Q_{IN,12} - |p(V_2 - V_1)| \quad (5)$$

NUMERICAL SUBSTITUTION:

EQU (4) $\longrightarrow$ $W_{OUT,12} = 0.2\ \text{MPa}(2. - 1.)\,m^3 \left(\frac{1\ N/m^2}{1\ Pa}\right)\left(\frac{1\ J}{1\ N \cdot m}\right) = 0.2$ MJ

EQU (5) $\longrightarrow$ $\Delta U_{12} = 2 - 0.2 = \boxed{1.8\ \text{MJ}}$