Water Tunnel Experiment

MAE 171A/175A

Objective:

Measurement of the Drag Coefficient of a Cylinder

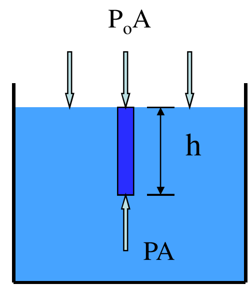
Measurement Techniques

- Pressure Distribution on Cylinder
- Drag from Momentum Loss Measured in Wake with laser Doppler Velocimetry - LDV

Review of Hydrostatic Pressures

- Demonstration of pressure increase with depth
- Weight of the column $W=mg=\rho Ahg$
- Weight is supported by the net pressure $=(P-P_o)A$

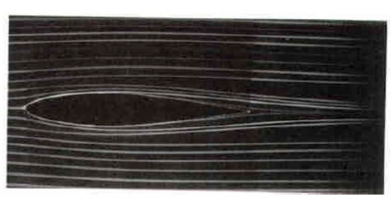
→ $P=P_o+\rho hg$

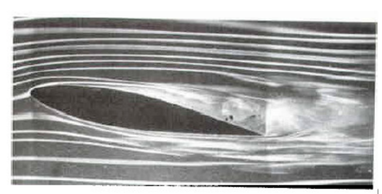


- Pressure increases with depth
- All points at a given depth are at the same pressure

Review of fluid motion

- The motion of fluid depends on the Reynolds No.
  - Laminar
  - Turbulent





- The laminar flow may be presented by streamlines
- When the fluid velocity is large or when the fluid encounters most obstacles, the flow becomes turbulent

Describing Fluid Behavior at Isothermal Conditions

- Conservation of Mass
- Newtons Second Law of Motion

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{w}) = 0$$

ρ- density
$\vec{w}$- velocity
t - time

$$\vec{\nabla} = \vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z}$$

Constant density:

$$\nabla \cdot \vec{w} = 0$$

Note: This is valid for unsteady flow

Newton's Second Law – Euler's Equation

$$\rho \frac{D\vec{w}}{Dt} = \rho \frac{\partial \vec{w}}{\partial t} + \rho(\vec{w} \cdot \vec{\nabla})\vec{w} = -\vec{\nabla} p + \rho \vec{F}$$

p – pressure
$\vec{F}$ - body force per unit mass

Assume body force is due to gravity, and it is conservative

$$\vec{F} = -g\vec{\nabla} z$$

g – acceleration due to gravity

Euler's Equation

Vector identity:

$$(w \cdot \nabla)w = \tfrac{1}{2}\vec{\nabla}(\vec{w} \cdot \vec{w}) - \vec{w} \times \nabla \times \vec{w}$$

Thus

$$\rho \frac{\partial \vec{w}}{\partial t} + \frac{1}{2}\rho \nabla(\vec{w} \cdot \vec{w}) + \nabla p + \rho g \vec{\nabla} z = \rho \vec{w} \times \nabla \times \vec{w}$$

Constant density

$$\frac{\partial \vec{w}}{\partial t} + \nabla\left[\frac{1}{2}\vec{w} \cdot \vec{w} + \frac{p}{\rho} + gz\right] = \vec{w} \times \nabla \times \vec{w}$$

Bernoullis Equation – Steady Rotational Flow

Consider Flow Along a Streamline

$$\left\{\vec{\nabla}\left[\tfrac{1}{2}\vec{w}\cdot\vec{w}+\frac{p}{\rho}+gz\right]=\vec{w}\times\vec{\nabla}\times\vec{w}\right\}\cdot d\vec{S}$$

Along a streamline

$$\tfrac{1}{2}\vec{w}\cdot\vec{w}+\frac{p}{\rho}+gz=const$$

Bernoullis Equation – Unsteady Irrotational Flow

$$\nabla\times\vec{w}=0 \qquad \vec{w}=\vec{\nabla}\phi$$

$$\nabla\left[\frac{\partial\phi}{\partial t}+\frac{1}{2}\vec{w}\cdot\vec{w}+\frac{p}{\rho}+gz\right]=\vec{w}\times\vec{\nabla}\times\vec{w}=0$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\vec{w}\cdot\vec{w}+\frac{p}{\rho}+gz=const$$

Applications of Bernoulli's Equation

- We Measure Flow Velocity Using Bernoulli's Eqn:

$$P_a + \frac{1}{2}\rho_a U_a^2 = P_b + \frac{1}{2}\rho_b U_b^2$$

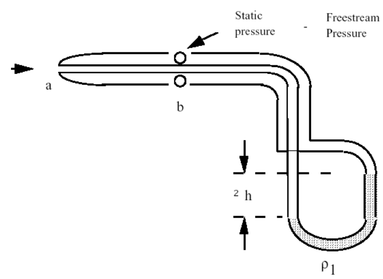
where:

$$U_a = 0, \; U_b = U_\infty$$

$$P_a - P_b = \frac{1}{2}\rho_\infty U_\infty^2 = q_\infty \quad \text{or} \quad P_a - P_b = \rho_1 \cdot \Delta h \cdot g$$

and:

$$U_\infty = \left(\frac{2(P_a - P_b)}{\rho_\infty}\right)^{1/2} \quad \text{or} \quad U_\infty = \left(\frac{2\rho_1 \Delta h \, g_e}{\rho_\infty}\right)^{1/2}$$



Pitot Probe: Measurement of Fluid Velocity

Δh is determined experimentally

Pitot tube was invented by a Frenchman Henry Pitot in 1732

Pressure Measurement Procedure

1.Calibrate the water tunnel test section by generating a plot of velocity versus motor frequency using the upstream pitot-static tube and Bernoulli's equation. Calibrate the operation of the water tunnel for motor frequencies from 15 to 40 Hz in 5 Hz increments. A calibration of the pressure transducer will be provided in the laboratory.

2. For velocities of 0.8, 1.4, and 1.8 m/sec plot the pressure coefficient as a function of angle around the cylinder on a single graph. The pressure measurements are located at the following angular positions.

1.	0
2.	22.5
3.	45
4.	56
5.	67
6.	78
7.	90
8.	101
9.	112
10.	123
11.	135
12.	157.5
13.	180

Before the next lab meeting:

Plot the pressure coefficient as a function of angle around the cylinder on a single graph. Compare the measured pressure coefficient distributions to that for the idealize flow around the cylinder, $C_p = 1\text{-}4 \sin^2(\theta)$, by plotting it along with the measured pressure coefficient.

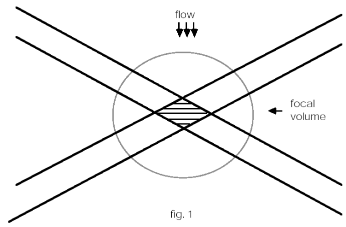
Part 2

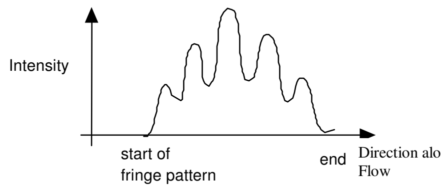
- Drag from Momentum Loss Measured in Wake with Laser Doppler Velocimetry - LDV

Laser Doppler Velocimetry

- It is a powerful technique used for highly accurate measurement of fluid velocity in liquid or gaseous flow
- It was developed in 1964 by Yeh and Cummins to measure laminar water flow
- Does not disturb the fluid flow
- The technique has been improved over the years and it has variety of applications

How does it work?



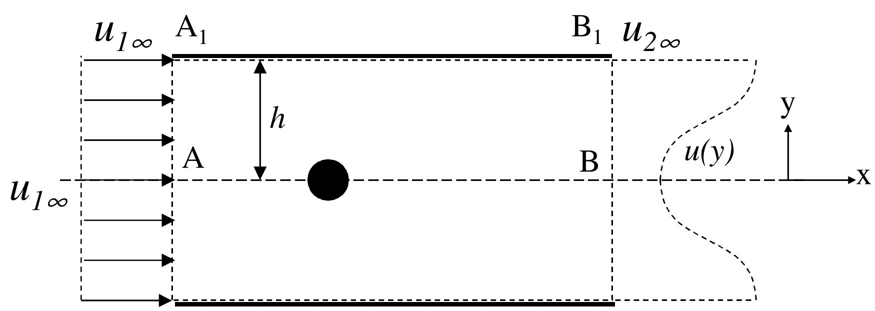


- A pair of laser beams are focused down to create interference fringes at the optical volume formed
- Due to the Gaussian intensity distribution of the laser beam, the fringes at the center of the focal volume will be brighter

Part 2

- Drag from Momentum Loss Measured in Wake with Pressure Array

Drag Coefficient



- incompressible

Drag Coefficient

<u>Mass Conservation</u>

<u>Cross Section:</u> (Rate of Flow)

$$AB = 0$$

$$AA_1 = \rho b \int_0^h u_{1\infty} dy = \rho b h u_{1\infty}$$

$$BB_1 = -\rho b \int_0^h u dy$$

$$A_1B_1 = -\rho b \int_0^h (u_{1\infty} - u) dy$$

Drag Coefficient

Equation of Motion – steady Flow:

$$\iiint [\nabla \cdot (\rho \vec{w}\vec{w}) = -\nabla p] dV + Drag$$

Reynolds Transport Theorem

$$\iiint \vec{\nabla} \cdot \vec{F} dV = \iint \vec{F} \cdot \vec{n} dA$$

$$\iint \rho \vec{w}\vec{w} \cdot \vec{n} dA = -\iint p\vec{n} dA + Drag$$

$$Drag = \iint \rho \vec{w}\vec{w} \cdot \vec{n} dA$$

If static pressure on AA_1, BB_1 are same

Drag Coefficient

Equation of Motion

Cross Section	$\int \rho \vec{w}\vec{w} \cdot n dA$
AB	0
AA_1	$\rho b h u_{1\infty}^2$
BB_1	$-\rho b \int_0^h u^2 dy$
A_1B_1	$-\rho b \int_0^h u_{1\infty}(u_{1\infty} - u) dy$

$$Drag = 2\rho b \int_0^h u(u_{1\infty} - u) dy$$

$$= 2\rho b u_{1\infty}^2 \int_0^h \frac{u}{u_{1\infty}} \left[1 - \frac{u}{u_{1\infty}}\right] dy$$

Drag Coefficient

Drag on Cylinder:

Per unit length (b = 1)

$$Drag = 2\rho b u_{1\infty}^2 \int_0^h \frac{u}{u_{1\infty}}\left[1-\frac{u}{u_{1\infty}}\right]dy$$

The Drag Coefficient C_D

The Drag coefficient C_D is defined by normalizing the Drag by the dynamic pressure $\frac{1}{2}\rho U_{1\infty}^2$ and the projected area of the cylinder $A_p = D\cdot(b=1)$ where D is the diameter of the cylinder and b=1 is the unit length of the cylinder. The Drag coefficient per unit length of cylinder is:

In equation (5) there are two methods of determining the ratio of $\left(\frac{U_{2\infty}}{U_{1\infty}}\right)$ either from eq. (1) or from the measured values of $U_{1\infty}$ and $U_{2\infty}$.

$$C_D = \frac{Drag}{\frac{1}{2}\rho U_{1\infty}^2 \cdot A_p} = \frac{Drag}{\frac{1}{2}\rho U_{1\infty}^2 \cdot D} \qquad (6)$$

This is measured using Pitot tube

Velocity Measurement Procedure

1. Using the LDV system reconfirm the velocity in the water tunnel versus motor rpm over the range of operation by plotting the results along with the previously measured velocities from the pitot probe analysis.

2. Use the same motor frequencies used previously in pressure distribution measurements to set water tunnel velocities of 0.9, 1.4, and 1.75 m/sec . Use the LDV system to measure the axial velocity and RMS velocity 6 cylinder diameters upstsream to 16 diameters downstream (use 0.5 diameter increments except downstream beyond 6 diameters use 1.0 diameter increments). Plot the velocity and the RMS velocity for each flow condition as a function of distance from the cylinder on separate graphs and compare with the velocity from the pitot probe calibration. Also, on a separate graph plot the RMS velocity normalized by the absolute value of the local mean velocity as a function of distance from the cylinder.

3. For water tunnel velocities of 0.9, 1.4, and 1.75 m/sec use the LDV system to measure velocity profiles transverse to the axis of the cylinder 6 diameters upstream of the cylinder. Compare to a uniform velocity profile. Is there an upstream influence of the flow around the cylinder?

Velocity Measurement Procedure

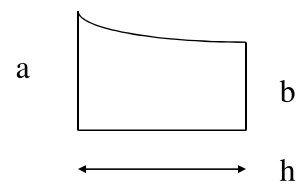
4. For water tunnel velocities of 0.9, 1.4, and 1.75 m/sec use the LDV system to measure velocity profiles transverse to the axis of the cylinder 8 and 16 diameters downstream of the cylinder.

5. Calculate the drag coefficient for the cylinder for water tunnel velocities of 0.9, 1.4, and 1.75 m/sec using the velocity profiles in item 4 and the analysis given by eq.(5) in the discussion section. Compare the measured ratio of $\left(\frac{U_{2\infty}}{U_{1\infty}}\right)$ with that calculated from eq.(1). You are required to calculate the error in the drag coefficient based on the error in the velocity measurements.

Trapezoidal Rule

- Direct way to calculate numerical integral is by breaking of the area under the curve into small known area squares and counting the squares
- Another way is to breakup the curve into straight line segments , therefore creating trapezoids and add up the areas of these geometric figures
- The area of a single trapezoid (with width h and height a and b)

Area=h(a+b)/2



Trapezoidal Rule

- Apply this rule to an integral I:

$$I = \int_a^b f(x)dx$$

- Approximate I with a finite difference:

$$I \approx \sum_{i=1}^{N-1} \frac{1}{2}\left[f(x_i) + f(x_{i+1})\right](x_{i+1} - x_i)$$

- Here i is an index that goes from the first data point (i=1) up to last data point available, i=N, x_i is the value of the independent variable at the i-th datapoint, and $f(x_i)$ is value of the function f at x_i .

h

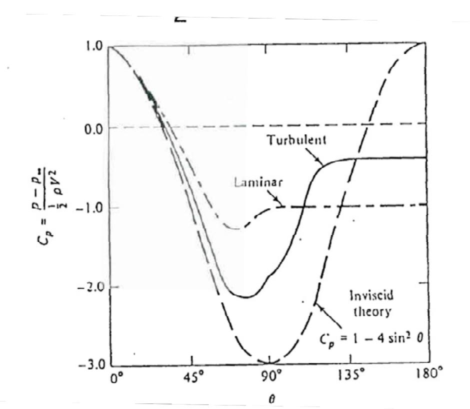
References

1. John D. Anderson, "Fundamentals of Aerodynamics", 2nd Ed., pp. 195-200 and pp. 228-236, McGraw Hill 1991.
2. L. E. Drain, "The Laser Doppler Technique", John Wiley & Sons 1980.
3. Harris Benson, "University Physics", Chapter 14, John Wiley & Sons 1991
4. http://clients.dedicatedconsulting.com/aerometrics/ldv.html#LDV

Effect of Finite Viscosity

- What is the Reynolds number?

The Reynolds number is the ratio of *inertial forces* , as described by Newton's second law of motion, to *viscous forces*



$$Re = \frac{\rho U L}{\mu}$$