MAE 171A
Mechanical Engineering Laboratory

Fracture mechanics and
viscoelastic response of polymers
(**time-dependent deformation)**

Lecture Notes

Outline

- ⇒Theoretical cohesive strength of materials
- ⇒Stress intensity factor
- ⇒Griffith fracture theory for brittle fracture
  - ॐDevelopment of stress intensity factor
- ⇒Introduction to polymers
  - ॐMechanical properties
- ⇒Viscoelastic response of polymers
  - ॐCreep and stress relaxation
  - ॐTemperature effects
- ⇒Laboratory experiments

Some disasters due to material failure

4-28-1988 After 89,090 flight cycles on a 737-200, metal fatigue lets the top go in flight.

Aloha Airlines 1988

Small section of roof ripped 12 cm

Resulting explosive decompression

tore off the entire top half of the aircraft

Challenger 1986

O-ring seal in the

right solid rocket booster failed

and caused a flame

SS Schenectady

1944 brittle fracture

- •2500 Liberty ships built
- •700 experienced severe structural failure
- •145 broke into 2 pieces

Reasons:
Flaws in welded joints
Low fracture toughness materials used

(Elastic energy stored in the bulk ~l^3)**/**(Energy to open a crack ~l^2) **~** l.
It is easy to break large objects and very difficult to break a small ones!

Recent disaster

Report suggests sand mounds stressed Minnesota bridge.

The New York Times (3/18, 2008) reports, "The Interstate 35W bridge over the Mississippi in Minneapolis collapsed after construction workers had put **99 tons of sand on the roadway directly over two of the bridge's weakest points**," a new report from the National Transportation Safety Board (NTSB) revealed. According to a report "Stress at one of the two weakest points was 83 percent more than it could have handled." However, the load would not have been excessive for a well-designed bridge, according to experts."

Constructive applications of fracture

1. To break into small pieces large diameter radioactive metallic vessels for recycling after atomic power stations will be closed

2. Fragmentation of minerals in gold mining (median size of particles in attritor is leveled at about 1 micron and has very low dependence on specific energy input

3. Utilization of high quality steel from military equipment (e.g., tanks)

Development of theoretical strength of a material based on different parameters

⇒Two ways to estimate strength

(1) Theoretical cohesive strength

Force necessary to break atomic bonds

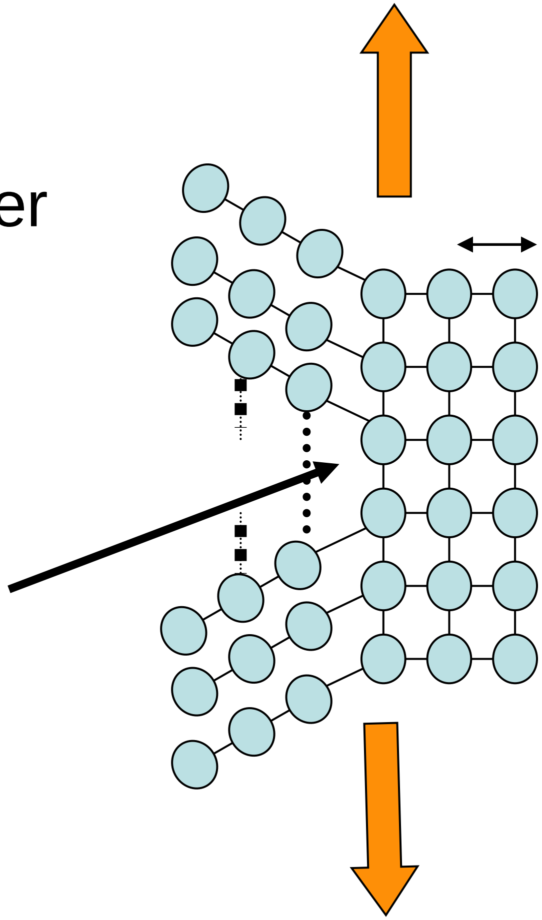
(2) Work of fracture to create new free surfaces

→Area under the stress strain curve

(1) Theoretical cohesive strength

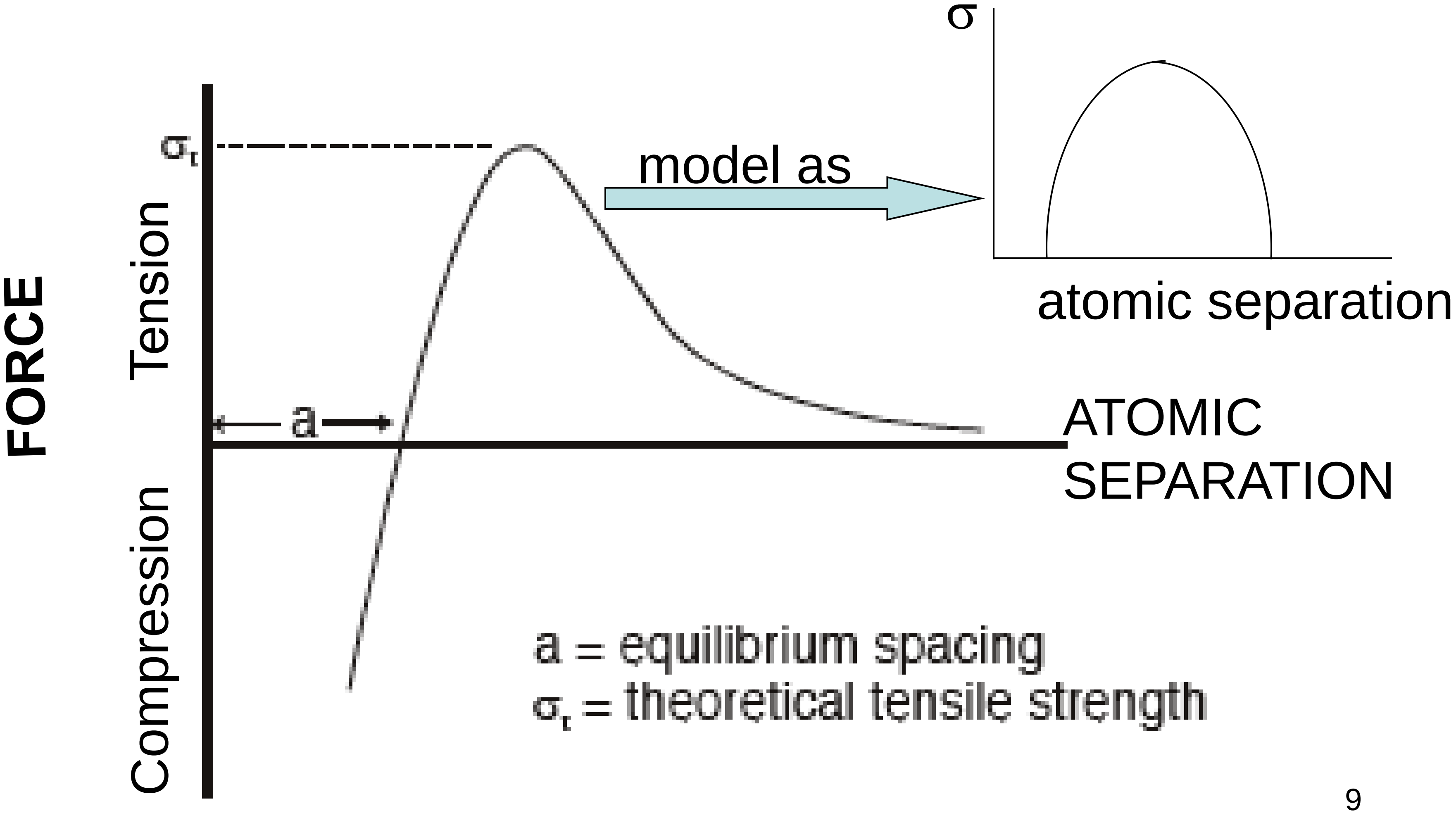
⇒First consider the theoretical cohesive strength of a material

- A material is held together by strong atomic bonds
- Consider a crack in the material
  - How much stress must be added to break the bond?



a_o = atomic spacing

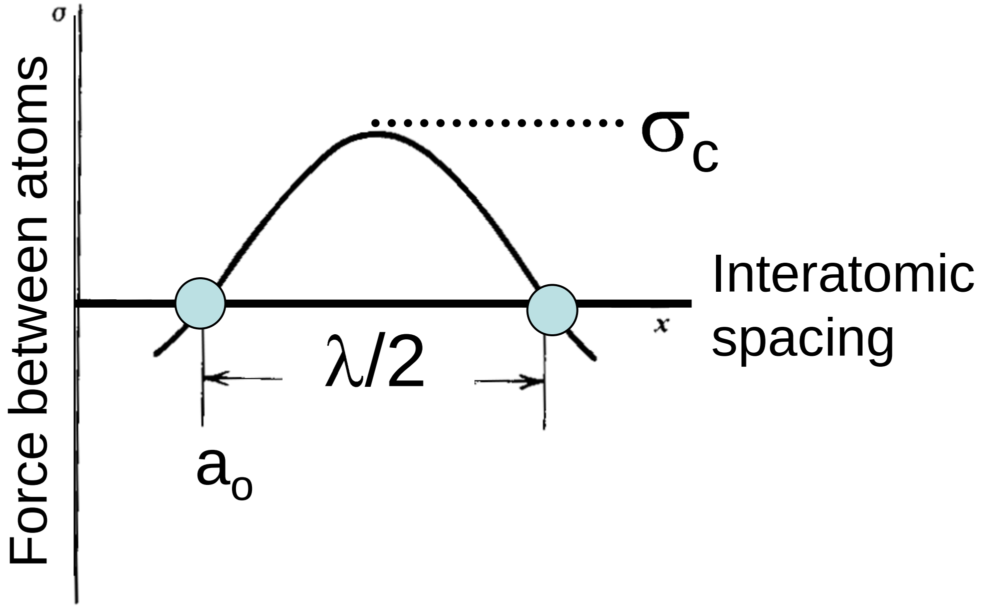
(1) Theoretical cohesive strength – *force (stress)-displacement curve*



a = equilibrium spacing
σ_t = theoretical tensile strength

(1) Theoretical cohesive strength

Consider that the cohesive strength of a material is the force required to separate two atoms (break bonds)



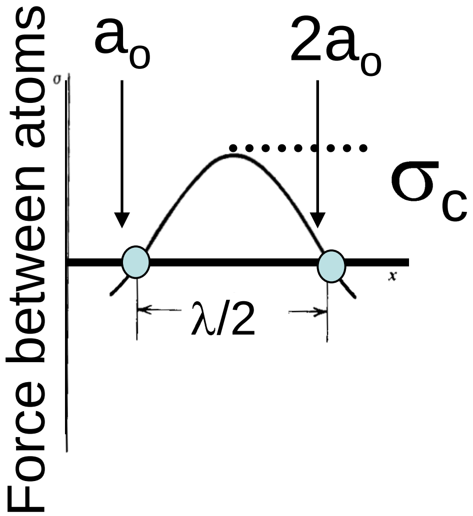
σ_c = cohesive strength
a_o = equilibrium atomic spacing
⇒no force on the atoms

Simplified force vs. atom displacement relationship to describe cohesive strength (fracture strength)

(1) Theoretical cohesive strength

Assume $\sigma = \sigma_c \sin\left(\frac{2\pi x}{\lambda}\right)$ and $\sin\left(\frac{2\pi x}{\lambda}\right) \approx \frac{2\pi x}{\lambda}$ for small values of $\frac{2\pi x}{\lambda}$,

thus $\sigma = \frac{2\sigma_c \pi x}{\lambda}$



The slope of the curve for small displacements

$$\frac{d\sigma}{dx} = \frac{2\sigma_c \pi}{\lambda}$$

(1) Theoretical cohesive strength

From other (continuum) point of view Hooke’s law must also apply in this region :

$\sigma = E\varepsilon$ and $\varepsilon = \dfrac{x}{a_o}$ where $\mathrm{a_o}$ is the lattice constant, x - displacement

$\lambda \sim 2a_o$

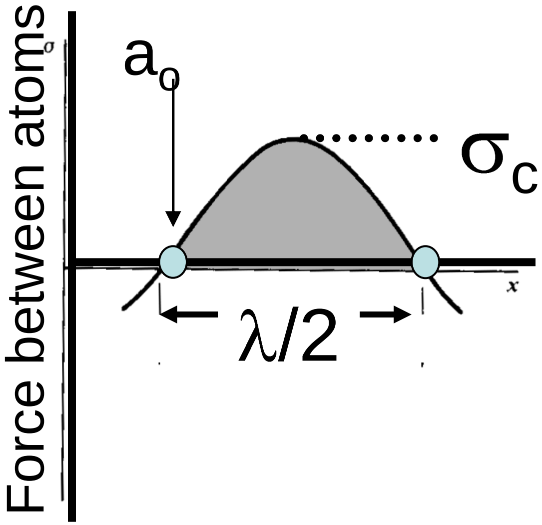
$\sigma = E\dfrac{x}{a_o}$ then $\dfrac{d\sigma}{dx} = \dfrac{E}{a_o}$, using equation from previous slide $\dfrac{E}{a_o} = \dfrac{2\pi\sigma_c}{\lambda}$

obtain $\sigma_\mathrm{c} = \dfrac{E}{\pi}$

(2) Work of fracture

⇒Another method to calculate the cohesive strength is to consider the work of fracture

⇒The area under the force displacement curve can be used to describe the work of fracture per atom



$$Work = \int_0^{\lambda/2} \sigma_c \sin\left(\frac{2\pi x}{\lambda}\right) dx$$

(2) Work of fracture

Solving the integral gives :

$$Work = \frac{\sigma_c \lambda}{\pi} = 2\gamma_s$$ where γ_s is the surface energy

(2 new surfaces created)

Using also previous equation for σ_c : $\frac{E}{a_o} = \frac{2\pi\sigma_c}{\lambda}$

get $$\sigma_c = \boxed{\sqrt{\frac{E\gamma_s}{a_o}}}$$

For most materials, values for γ_s and a_o give

$$\boxed{\sigma_c = E/10}$$

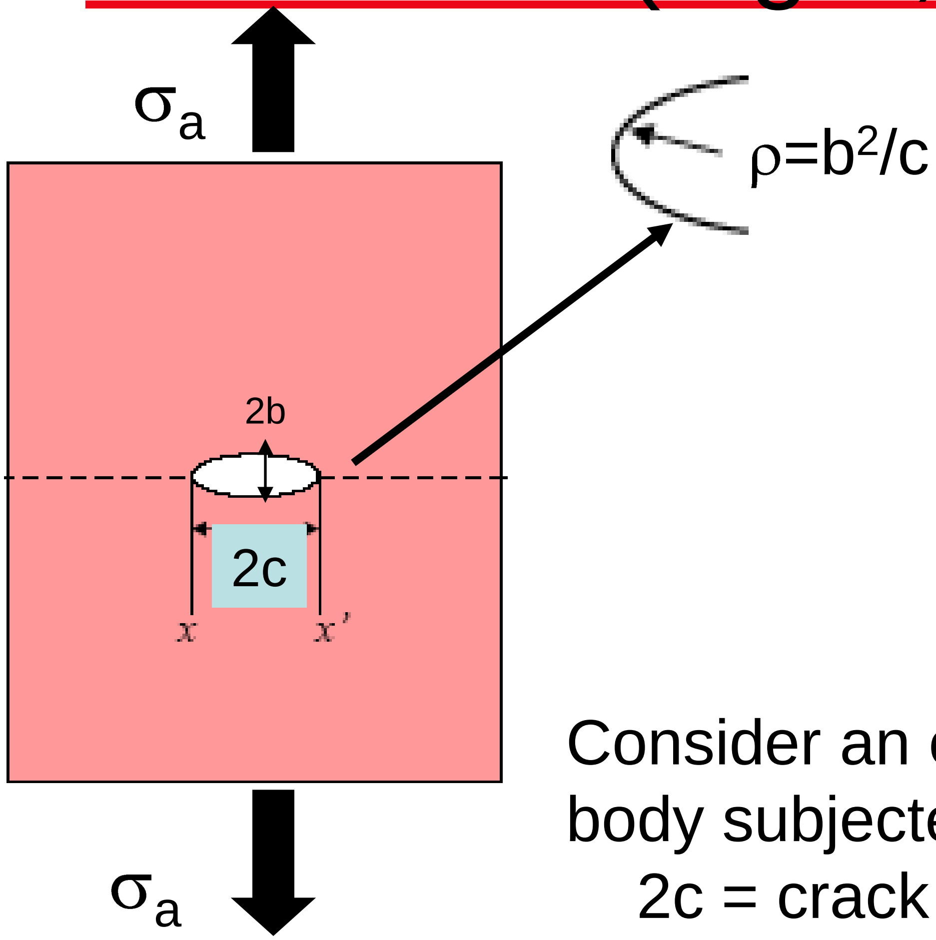
Theoretical vs. experimental values

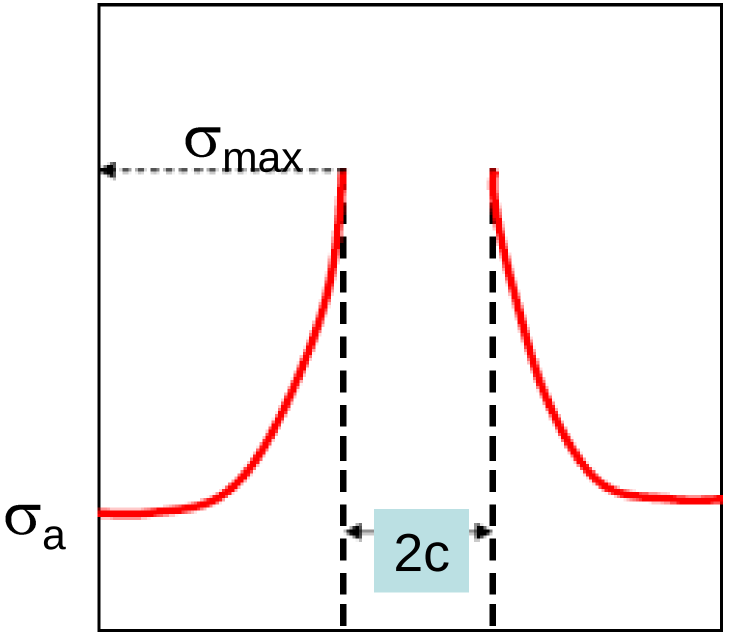
material	**E (GPa)**		**UTS (GPa) predicted**	**UTS (GPa) experimental**
glass	70		7	0.006
aluminum oxide	400		40	2
stainless steel	200		20	0.9
aluminum	70		7	0.5
nylon	70		7	0.05

Why are theoretical values orders of magnitude much greater than experimental values (except for freshly made glass fibers which strength approached the theoretical tensile strength of the order of one-tenth the elastic modulus)? The fiber strength was increased by polishing.

This is the problem Inglis and Griffith tackled

Stress concentration factors (Inglis, 1913)





Consider an elliptical crack in the center of a body subjected to an applied tensile stress σ_a

2c = crack length

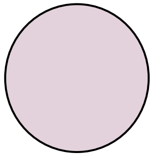
2b = crack width

σ_{max} = max stress at the end of major axis

Stress concentration factors

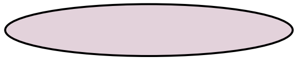
The plate is subjected to uniform in-plane tensile stresses perpendicular to the major axis of the elliptical notch. The maximum tensile stress $\sigma_{\max}$ would occur at the ends of the major axis of the elliptical notch

$$\frac{\sigma_{\max}}{\sigma_a} = 1 + \frac{2c}{b} \quad \text{(stress concetration factor for the notch)}$$



c/b = 1

$\sigma_{max} = 3\sigma_a$



c/b = 2

$\sigma_{max} = 5\sigma_a$

The radius of curvature for *elliptic* crack $\rho = \dfrac{b^2}{c}$

$$\boxed{\sigma_{\max} = \sigma_a \left(1 + 2\sqrt{\frac{c}{\rho}}\right)}$$

Depends on the shape of the crack (relative value of c/b or c/ρ)

c/b = 16

$\sigma_{max} = 33\sigma_a$

Stress concentration factors

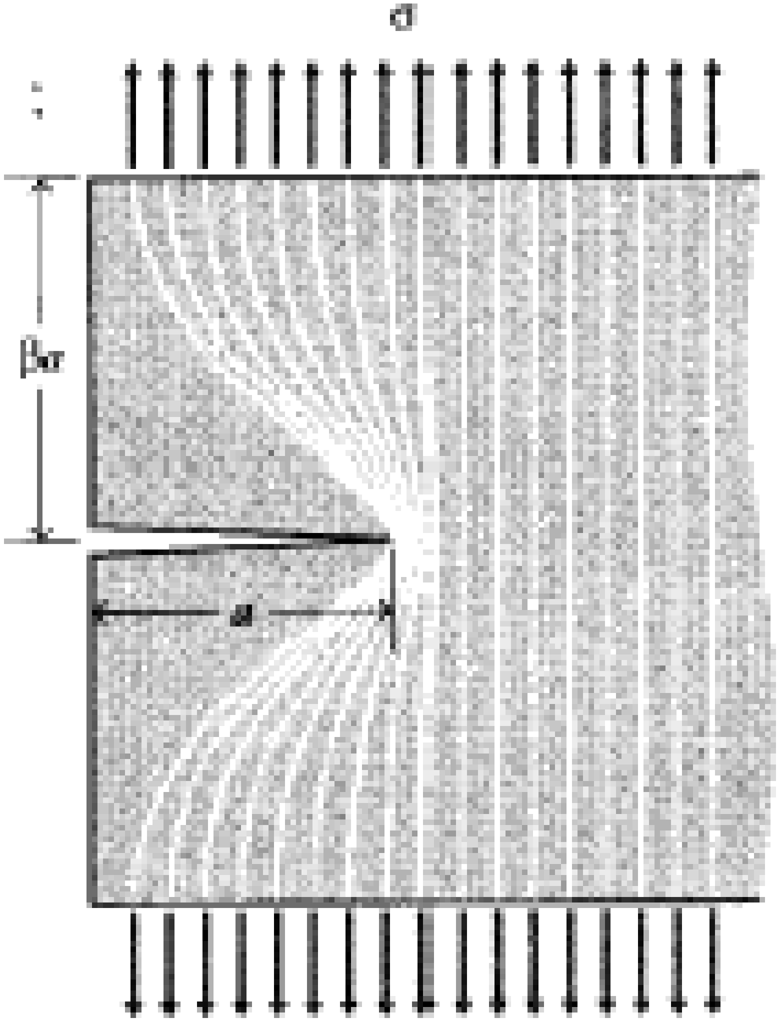
For sharp notches $(c >> \rho)$ $\sigma_{\max} = 2\sigma_a \sqrt{c/\rho}$

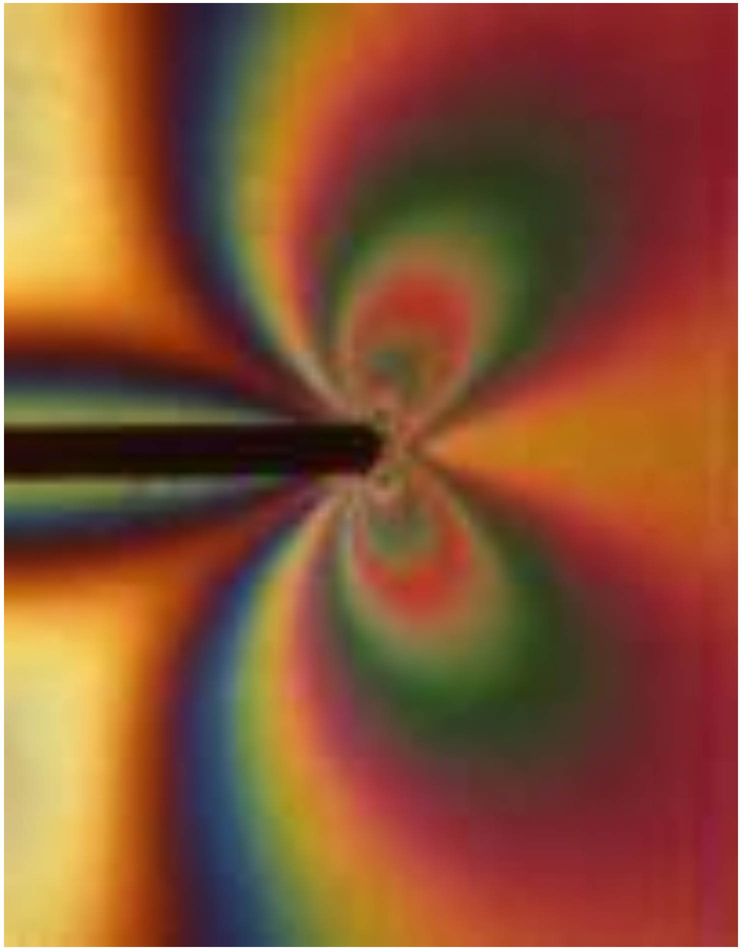
$$2\sqrt{c/\rho} \equiv \mathrm{k_t} = \text{stress concentrat ion factor}$$

$$\sigma_{max} = k_t\sigma_a$$

Fracture stress should approach zero as the radius of curvature is reduced to zero. Experimental data show that the stress required to produce fracture actually approaches a constant. Thus the maximum principal stress criterion for failure has limiting validity. (R.P. Wei, Fracture Mechanics, Cambridge, Univ. Press., 2010).

Stress concentration around a crack





Summary of Inglis formulation

⇒Showed that:

- Force applied to ends of an elastic plate would produce locally increased tensile stresses at the tip of a crack
    - may exceed the elastic limit of the material and lead to the propagation of the crack
- Increase in the length of the crack exaggerates the stress even more, such that the crack would continue to spread
- Small crack tip radii increase the stress at the crack tip
- The ***shape*** of the crack ***rather than the scale*** was important in determining the stress concentration

Problems with Inglis formulation

- ⇒The stress concentration factor, $k_t = \sigma_{max}/\sigma_a$
  - ॐHas no dependence on the crack scale
    - →Only on the ratio of c/ρ
- ⇒Even if local failure was initiated it does not mean that crack will propagate because crack propagation requires energy
- ⇒How can we approach a design problems so that we know the maximum stress a material can withstand, given a flaw size distribution?

Griffith fracture theory

(analysis of the equilibrium and stability of cracks)

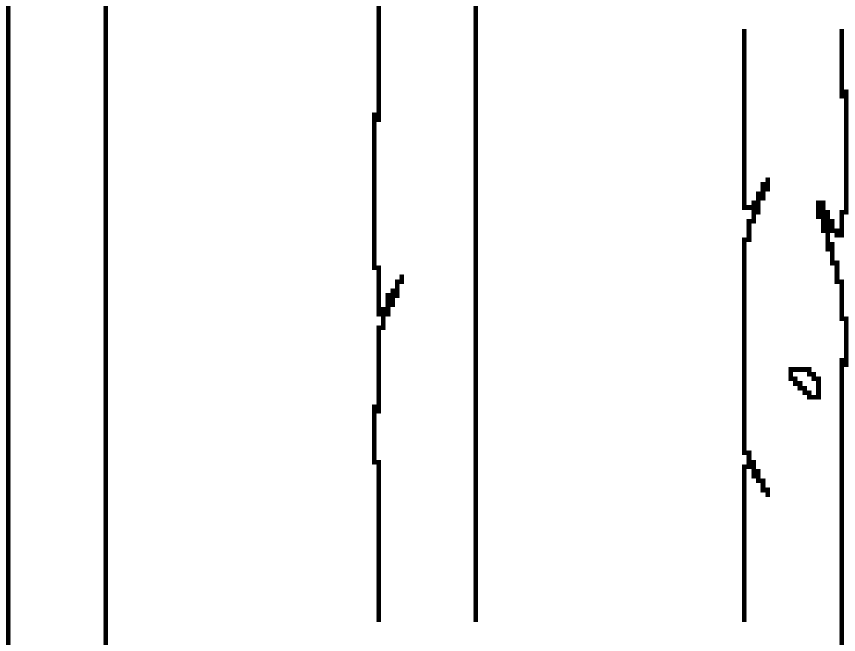
- ⇒ Tensile strength of freshly drawn glass fibers was much greater than that of old or aged fibers
- ⇒ Tensile strength decreased with increasing length of fiber
  - ॐ Volume effect
  - ॐ Same as da Vinci found
- ⇒ Tensile strength varied widely from sample to sample
- ⇒ A.A. Griffith, “The phenomena of rupture and flow in solids”, *Phil. Trans. Royal Soc.*, **221** (1921) pp. 163-198.

Alan Arnold Griffith
1893-1963

Griffith fracture theory

⇒Griffith concluded that:

- Fibers are weakened by microscopic flaws on the surface or interior of the fiber
  - Mechanical behavior will be dependent on presence of many small scale cracks
- Analysis can be extended to more general case of any brittle material



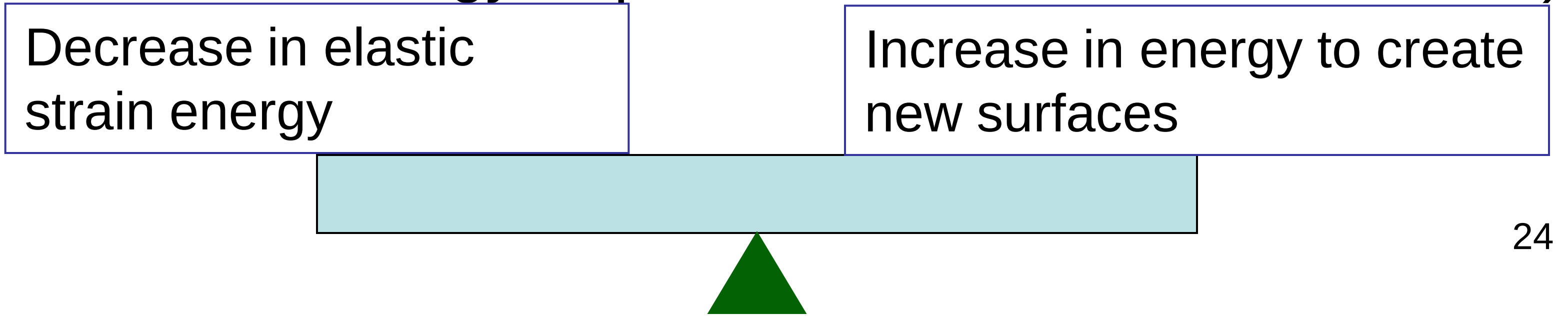
Griffith theory of brittle fracture

(new approach based on analysis of stability of crack)

⇒ When a crack propagates

  - There is a release of elastic strain energy
    - Decrease in energy

- During the crack extension process
  - New surfaces created at the faces of the crack
    - Increase of energy
- Griffith performed an energy balance (a defect would grow when the elastic energy released by the growth of the defect exceeded the energy required to form the crack surfaces)



Model of Griffith crack analysis

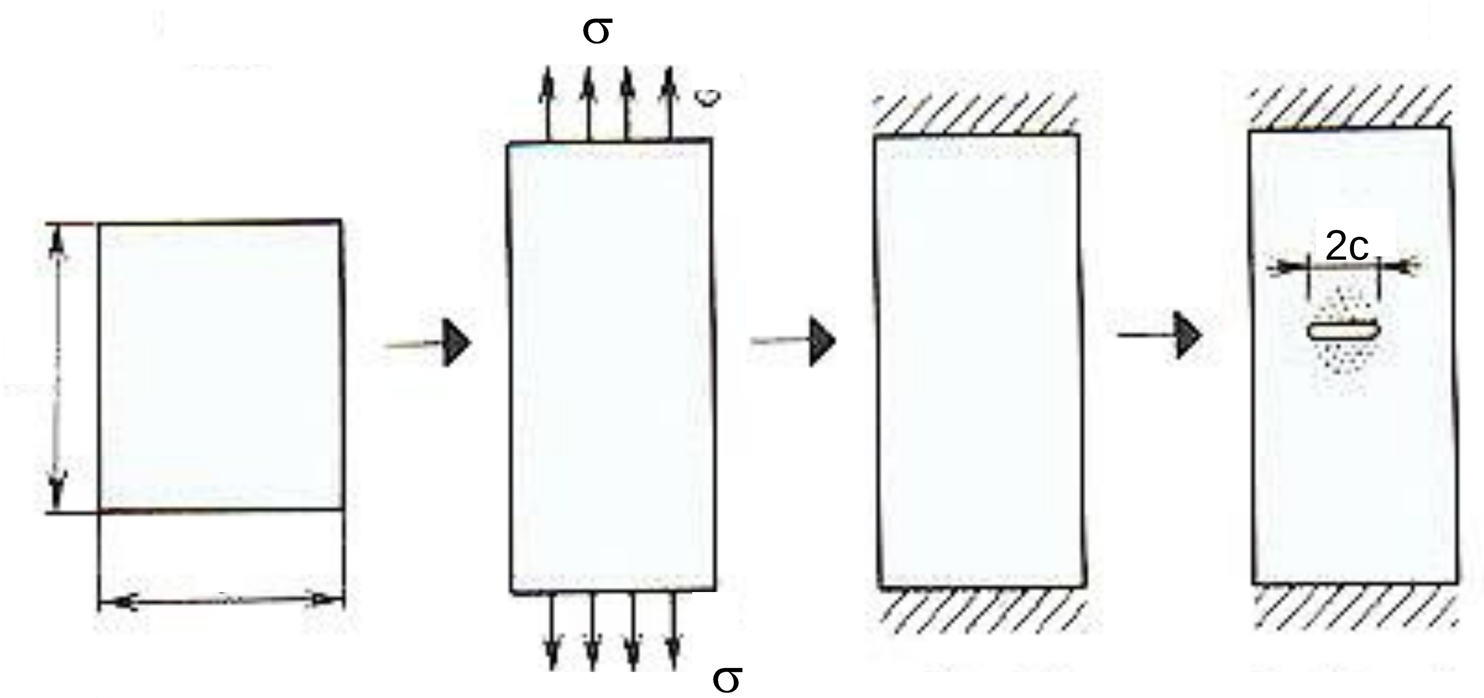


Plate
(thickness has a unit length)

extended

and rigidly fixed,
elastic energy is stored

unloading at
regions adjacent
to a growing
crack

Griffith theory of brittle fracture - plane strain condition

surface energy = $U_s = 4c\gamma_s$

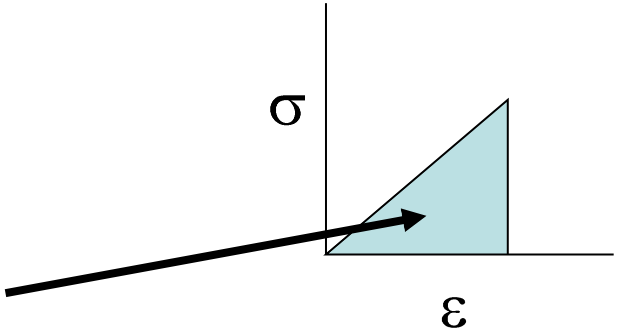
The decrease in elastic strain energy U_{SE}, for generalized plane stress, is given by: (for plain strain the numerator is modified by $(1-v^2)$ which usually for simplicity is not included in the discussion)

$$U_{SE} = -\frac{\pi\sigma^2 c^2}{E}$$

total energy $= U_T = U_s + U_{SE}$

Elastic strain energy given by area under σ - ε curve

Area = $\sigma\varepsilon/2$



Griffith theory of brittle fracture

$$U - U_0 = U_T = U_{SE} + U_S = -\frac{\pi\sigma^2 c^2}{E} + 4c\gamma_s$$

$U\,(U_0)$ – potential energy of body with (without) crack,

The maximum energy (equil.cond.) occurs when $\dfrac{dU_T}{dc} = 0$

The equilibrium is unstable

Griffith equation

Taking the derivative and setting $= 0$ yields $\sigma_c = \sqrt{\dfrac{2\gamma_s E}{\pi c}}$

σ_c is the critical stress needed for the crack to propagate: the crack will grow if $\sigma > \sigma_c$

How does this compare with experimental values?

- ⇒Griffith calculated γ_s for glass knowing:
  - $E = 6.2 \times 10^{10}$ N/m^2
  - $c = 0.001$ m
  - $\sigma_c = 8.3 \times 10^6$ N/m^2
    - → $\gamma_s = 1.75$ J/m^2
- ⇒Griffith measured γ_s as a function of T (between 745 and 1110 C) and extrapolated to RT
    - → $\gamma_s = 0.54$ J/m^2
- ⇒This is in excellent agreement despite three times difference!
  - Why? Because surface energies usually are measured within an order of magnitude

Fracture toughness

⇒For Griffith failure,

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi c}} \quad \text{then} \quad \sigma_c\sqrt{c} = \sqrt{\frac{2E\gamma_s}{\pi}} = \text{material property}$$

$K \equiv Y\sigma\sqrt{c}$ = stress intensity factor

$\boxed{K_{IC} = Y\sigma_c\sqrt{c}}$ where K_{IC} = fracture toughness under plane strain conditions

↑

refers to Mode I fracture

Y = geometrical constant = function of specimen and crack geometry

Fracture toughness

Fracture toughness characterizes the ability of a material to resist brittle fracture ***when a crack is present***

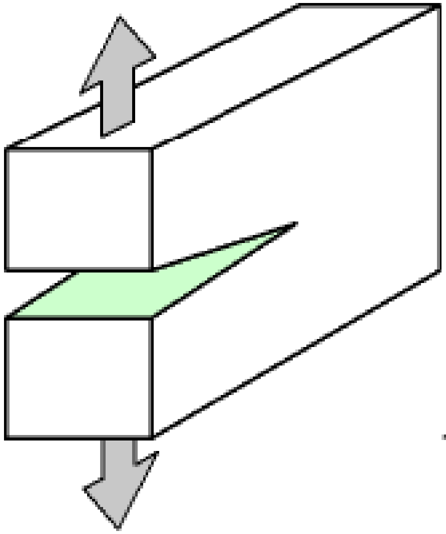
K_{IC} is a materials property (like density, elastic modulus, etc.) and has units **MPa√m.** It can not be calculated based on other mechanical properties and must be measured experimentally.

I refers to Mode I loading
c refers to ‘critical’ – where the material will fail

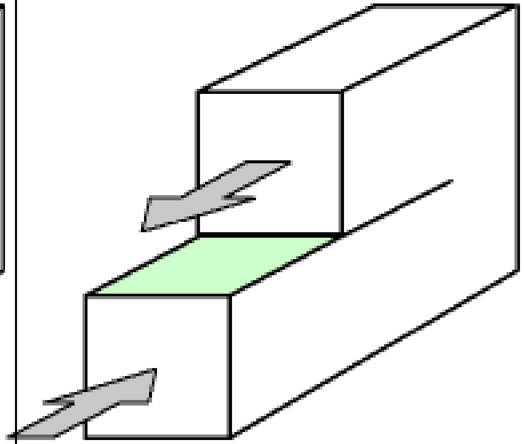
if you know K_{IC} and yield stress or TS, you can calculate the maximum flaw size that can be tolerated before material fails

The geometrical constant Y has many complex formulations

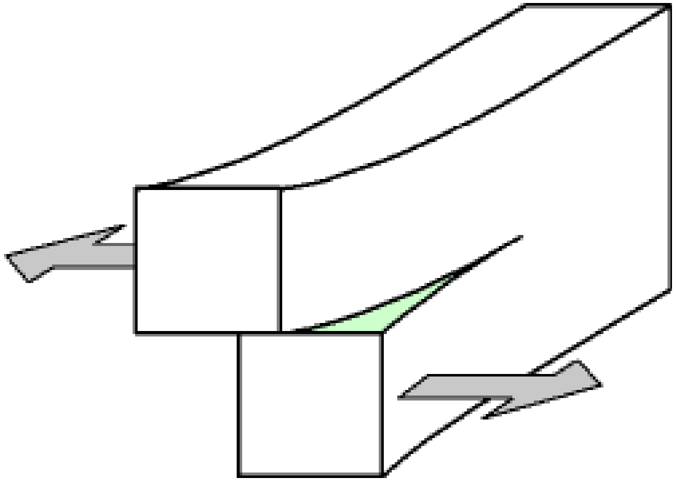
Fracture modes



Mode I Tension,
this fracture mode
we study in 171A

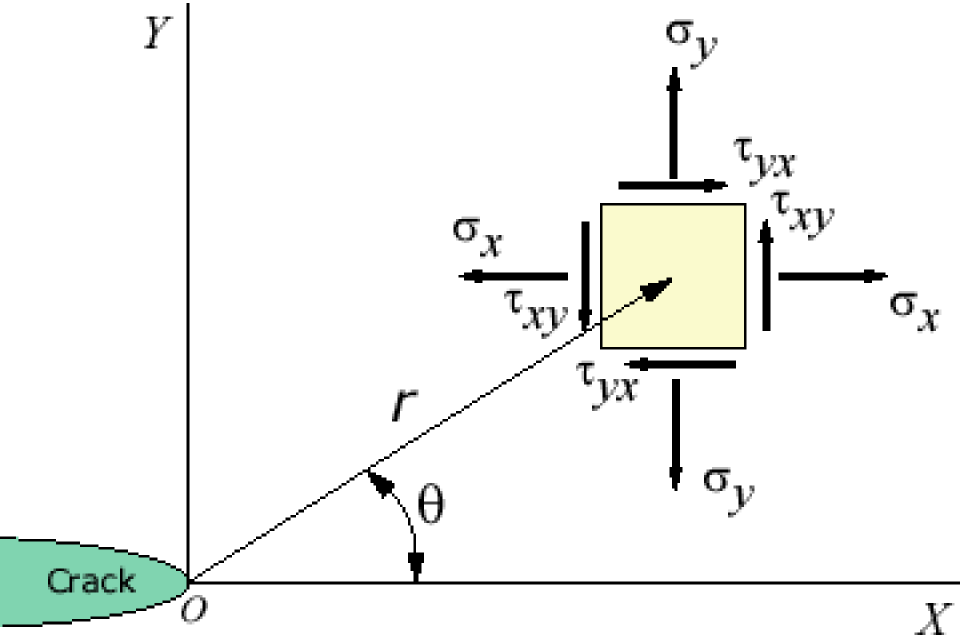


Mode II
In-plane shear
sliding



Mode III
Out-of-plane shear
tearing

Stress field around Mode I tensile loading



$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

k_t, K and K_{IC}

- ⇒ The stress concentration factor, k_t
  - ॐ $k_t = \sigma_{tip}/\sigma_{appl}$
    - → Ratio of stress at crack tip to applied nominal stress
    - → Can be approximated as ratio of $2\sqrt{(c/\rho)}$
- ⇒ The stress intensity factor, K_I in Mode I
  - ॐ It is a scale factor used to define the magnitude of crack-tip stress field
- ⇒ The fracture toughness, K_{IC}
  - ॐ a materials property, $K_I = K_{IC}$ at the onset of crack growth. If the specimen is thickness B is much greater than the plastic zone size $(B \geq 2.5\ (K_{IC}/\sigma_y)^2$ then abrupt fracture will occur when the crack-tip stress intensity factor reaches the plane strain fracture toughness K_{IC} .
    - → Subscript I refers to mode one, *plane strain* conditions
- ⇒ These k_t, K_I, and K_{IC} values should not be confused

Values for yield strength, elastic modulus and fracture toughness K_{IC}

	Yield Str.	**E**	**KIC**
	MPa	**GPa**	**MPam$^{1/2}$**
4340 steel	1470	200	46
Maraging steel	1730	200	90
Ti-6Al-4V	900	114	57
2024-T3 Al alloy	385	72.4	26
7075-T6 Al alloy	450	71.7	24

- **Fracture toughness is not proportional to yield strength or elastic modulus**
- **It may increase or decrease with increasing yield strength**

Fracture control design philosophy
(R.W. Hertzberg, Deformation and fracture mechanics of engineering materials, 1989)

Fracture toughness K_{IC}, design stress (σ) and crack size (a) control the conditions for fracture. For example, the fracture condition for an infinitely large cracked plate is

$K_{IC} = \sigma (\pi a)^{1/2}$, where

K_{IC} is determined by selection of a suitable material, e.g., Al alloys used in aircraft industry due to their high strength and low density;

σ is a design stress, e.g., determined by aircraft's payload capacity;

a is allowable flaw size or NDT flaw detection.

You must first decide what is most important about your component design: certain material properties, the design stress or the flaw size that must be tolerated for safe operation. Once any combination of two of these variables is defined, the third parameter is fixed.

For example, having fixed K_{IC} and σ, the allowable flaw size is defined by above equation and beyond the control of the aircraft designers. This allowable flaw size that can be tolerated by the material under the applied stress must be larger than the size of "hidden" crack (e.g., diameter of the rivet head covering the hole, usual source of high stress concentration in aircraft components).

Polymers

⇒Polymers

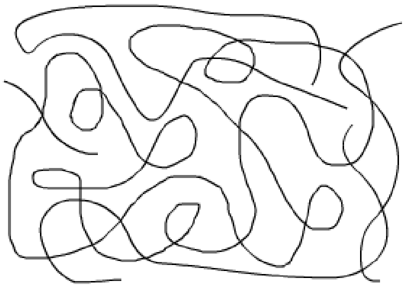
- Poly (many) mer (unit)
  - A polymer unit (mer) is typically composed of an organic group consisting of mainly C, H, and O atoms
  - Strong atomic bonds (covalent) hold these atoms together in the unit
    - These units are connect together to form large macromolecules in long chains composed typically of > 10,000 mer units (degree of polymerization)
- Macromolecules can be held together with *weaker hydrogen or van der Waals forces (thermoplastics*)
  - Can be recycled
- Macromolecules can be held together by *crosslinking (thermosets)*
  - Cannot be recycled
- The polymers used in this lab are thermoplastics

Natural polymers

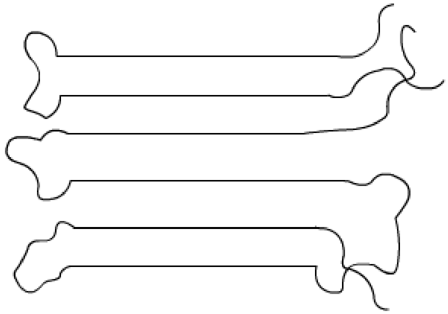
- ⇒ Spider silk fibers
  - Made up of intertwined proteins
    - Tensile strength ~ 1.6 GPa
      - Strongest naturally occurring
      - A strand long enough could circle globe
- Silk worm
  - Polypeptide
    - Synthetically called nylon
- Rubber
- Cellulose
- Chitin (polysaccharine)
  - Found in crustateons
    - → Hard, insoluble, flexible
      - Still can’t reproduce it In the lab

Structure of polymers

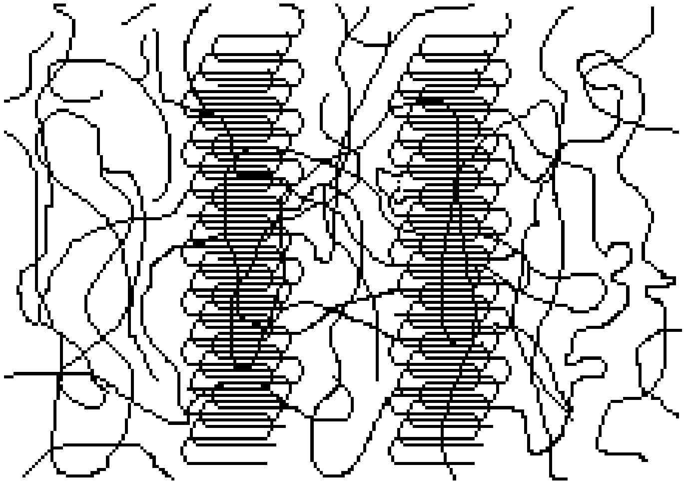
- Polymers have an amorphous structure
  - Long macromolecules tangled with each other
- Crystalline regions have orientation to the macromolecules
  - Orientation can result from tensile stress
    - Alignment of the long macromolecules
- Most polymers consist of amorphous and crystalline regions



amorphous

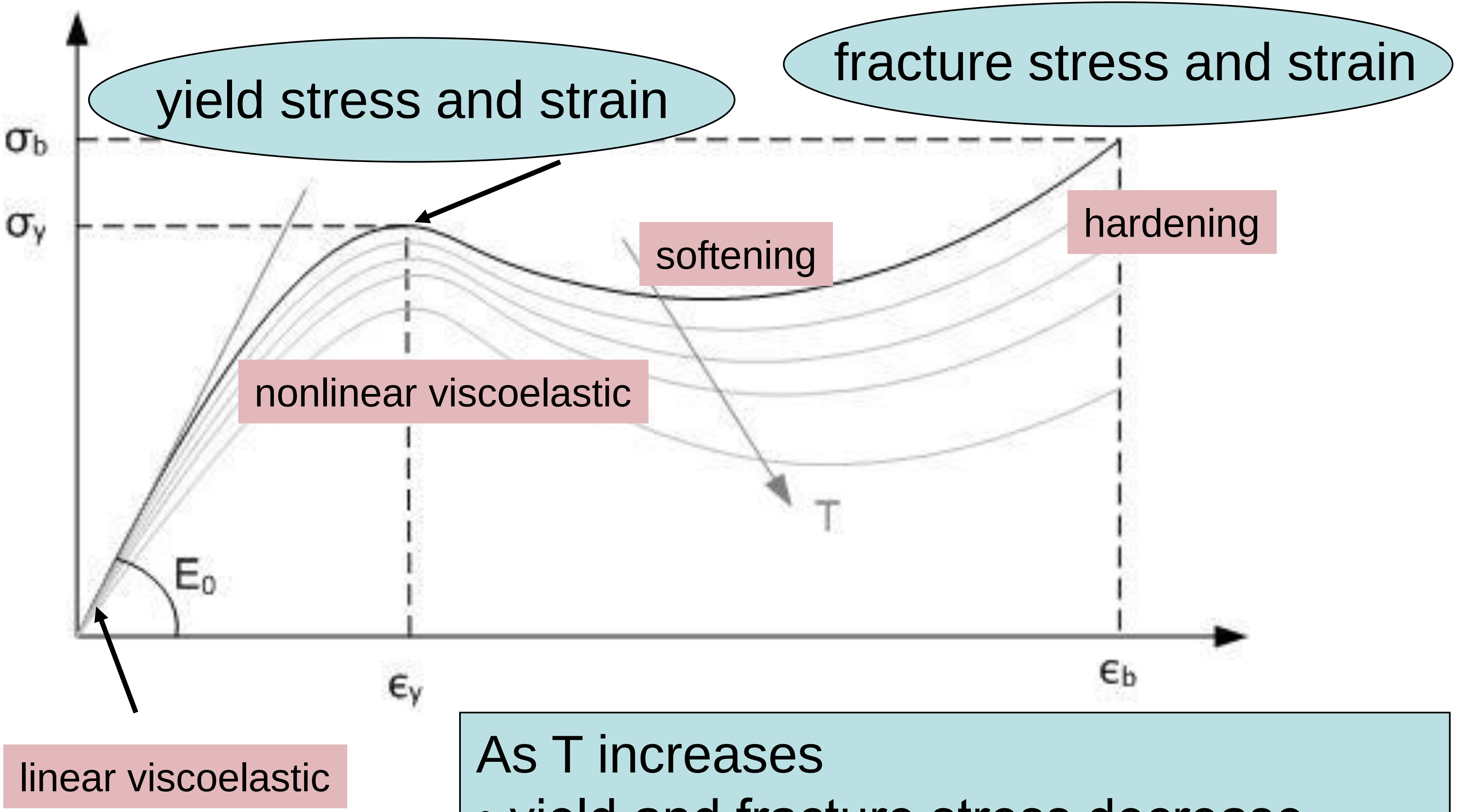


crystalline



amorphous + crystalline

Thermoplastics - stress/strain curve

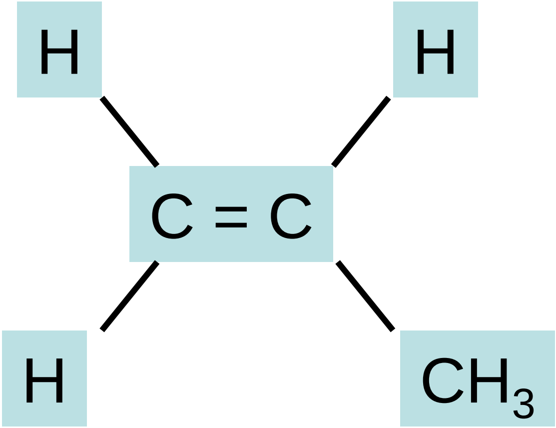


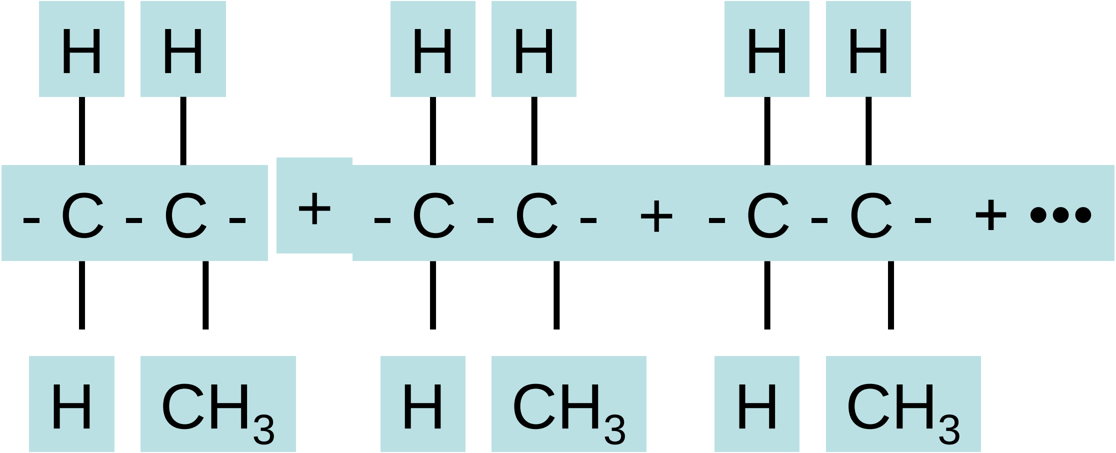
As T increases
- yield and fracture stress decrease
- elastic modulus at low strain decreases

Polypropylene

⇒Propylene (gas at RT) has the molecular formula C_3H_6

During polymerization, the double C = C is broken, leaving an unattached bond on both C atoms, creating the mer unit




$$-\underset{H}{\overset{H}{C}}-\underset{CH_3}{\overset{H}{C}}- + -\underset{H}{\overset{H}{C}}-\underset{CH_3}{\overset{H}{C}}- + -\underset{H}{\overset{H}{C}}-\underset{CH_3}{\overset{H}{C}}- + \cdots$$

These units link up to form a long chain. Other chains are formed to form a the solid polymer

Polypropylene (PP)

⇒Molecular formula $(C_3H_6)_n$

⇒Used for food packaging, textiles, reusable containers (Rubbermaid®)

5

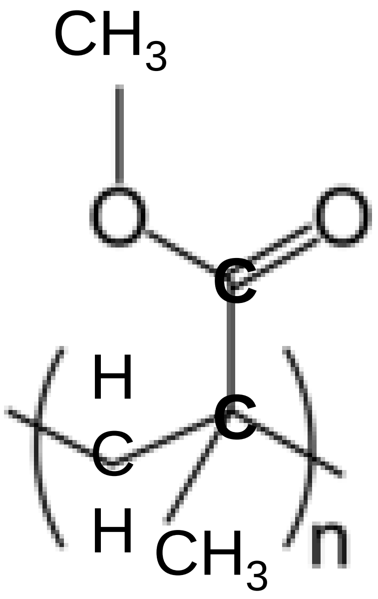
PP

⇒T_g = -20˚C

Packaging for DVDs

Polymethyl methacrylate (PMMA)

- ⇒Molecular formula $(C_5O_2H_8)_n$
- ⇒Trade names Lucite®, Plexiglas®
  - ॐUsed as an alternate to glass
- ⇒Rigid, impact resistant
- ⇒T_g = 105˚C



PMMA (‘acrylic’ glass)

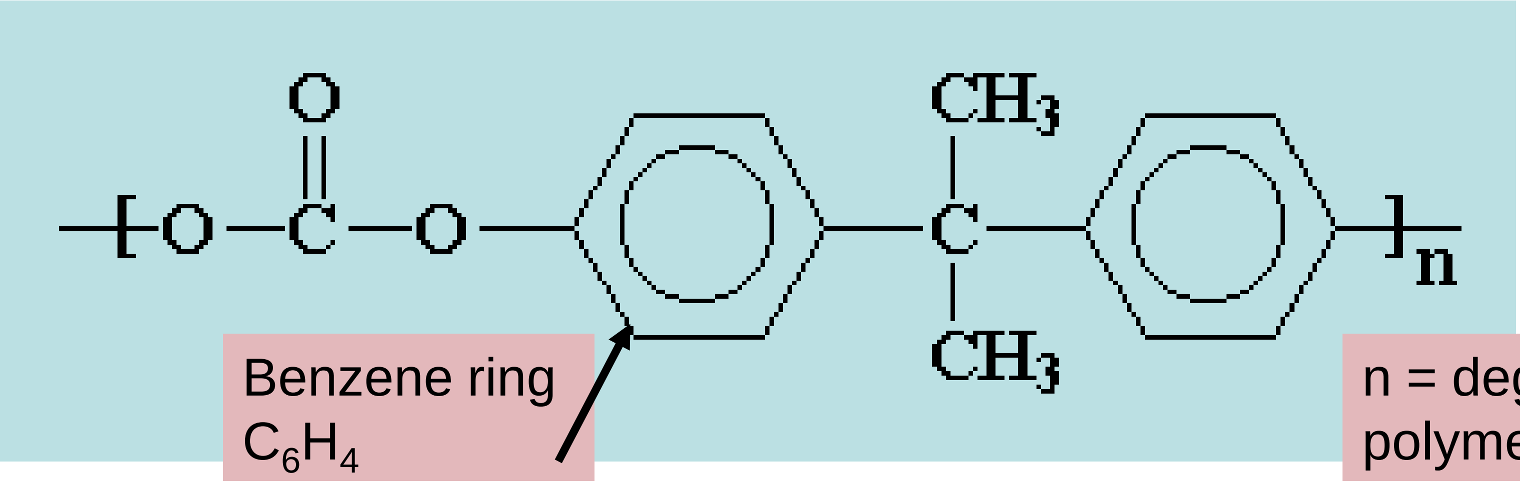
Ithaa -

worlds first PMMA glass undersea restaurant.
5 m below sea level.

Hilton Maldives Resort & Spa

Polycarbonate (PC)

- Molecular formula $(C_{16}H_{14}O_3)_n$
- Trade name Lexan®
- Used for glasses lens, DVDs, compact disks
  - Impact resistant - used for bullet-proof glass
- Stronger, more expensive than PMMA
- T_g = 150˚C

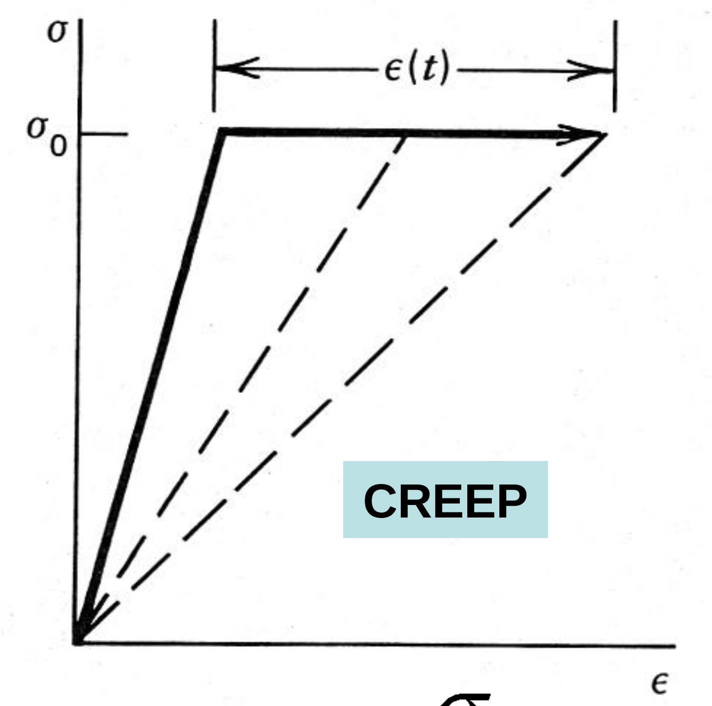


Benzene ring C_6H_4

n = degree of polymerization

Polycarbonate applications

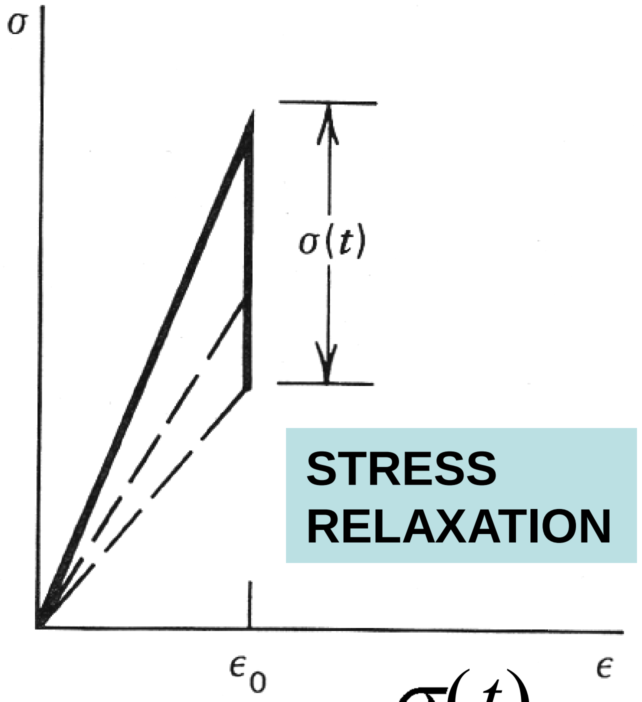
Viscoelastic response of polymers



$$E_c(t) = \frac{\sigma_o}{\varepsilon(t)}$$

Time dependent strain

Load constant



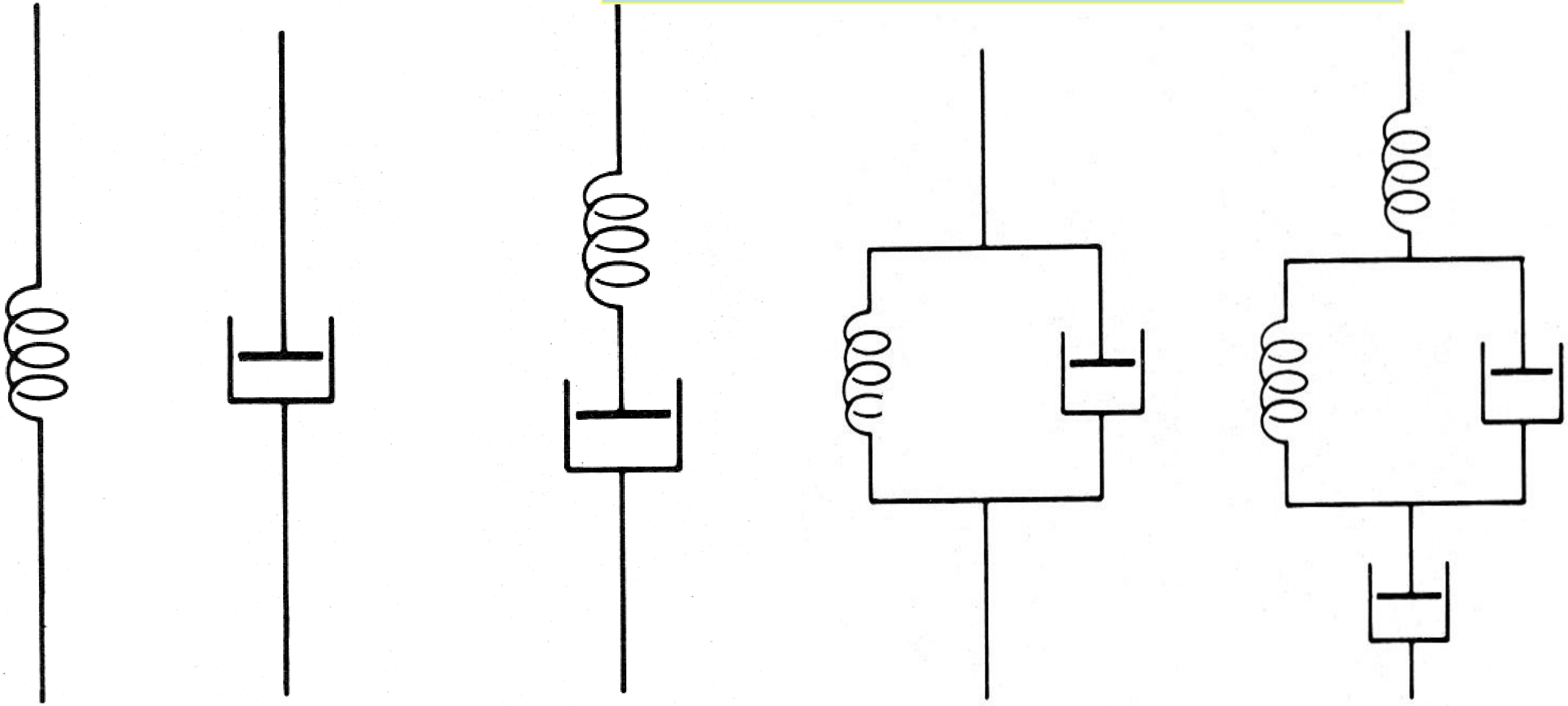
$$E_r(t) = \frac{\sigma(t)}{\varepsilon_o}$$

Time dependent stress

Elongation constant

Viscoelastic response - mechanical analogs (elements have zero mass)

Viscoelastic response models



Spring
(purely elastic)

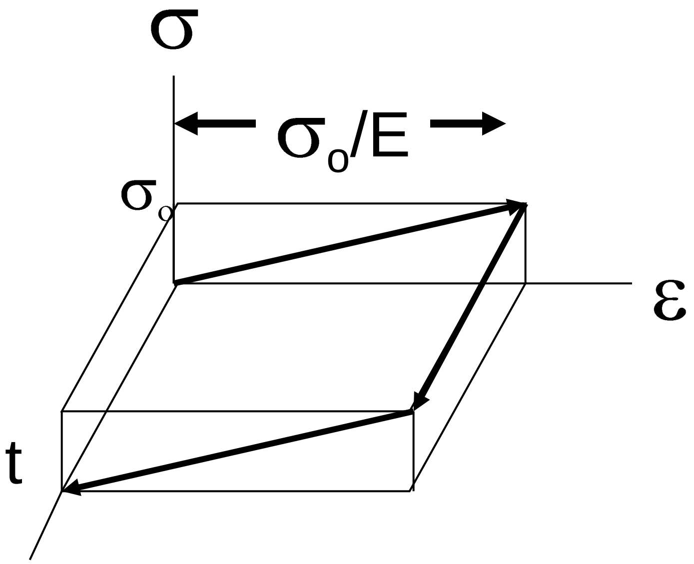
Dashpot
(purely viscous)

Maxwell
model

Voight
model

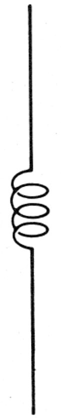
Four element
model

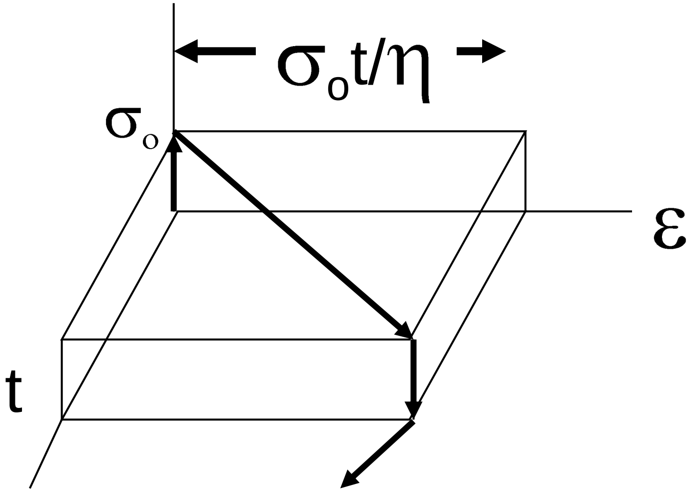
Spring and dashpot



⇒ Spring (elastic deformation)

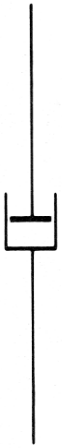
- ॐ$\sigma = E\varepsilon$, $\varepsilon=\sigma/E$
- ॐor $\gamma = \tau/G$
  - →γ = shear strain
  - →τ = shear stress
  - →G = shear modulus



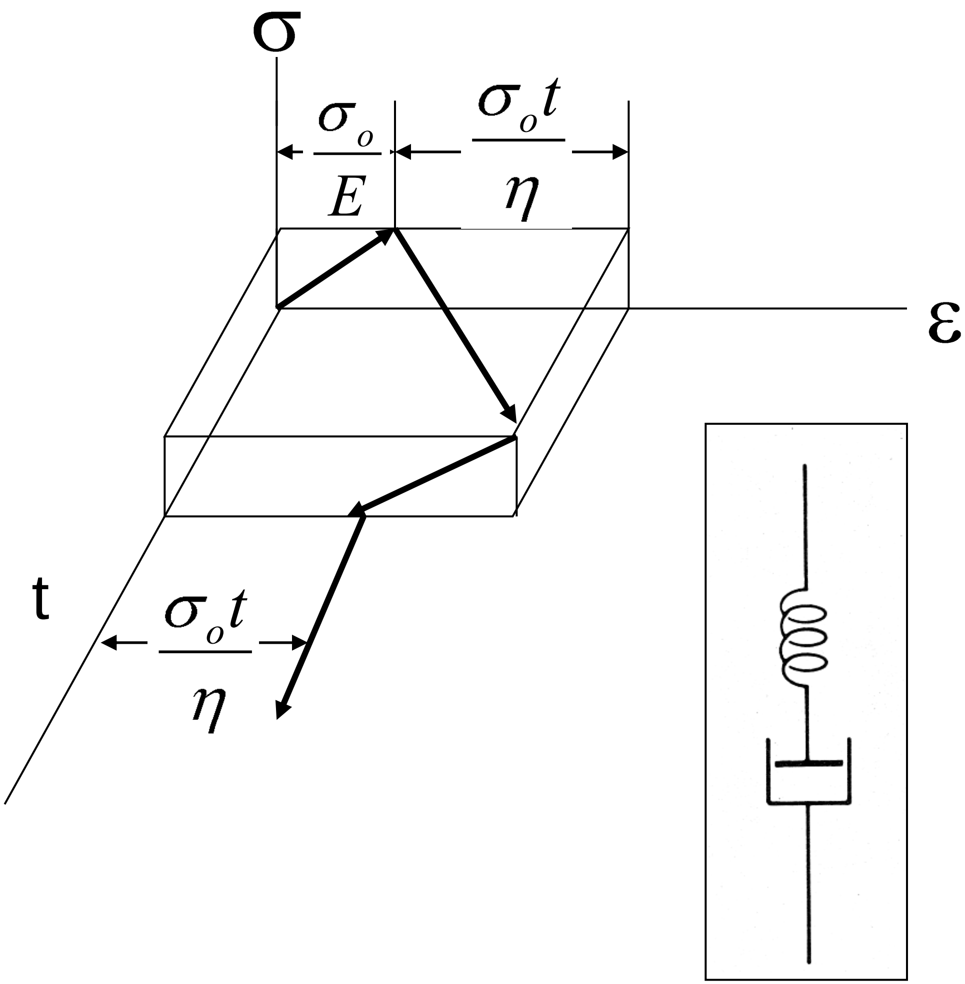


⇒ Dashpot (viscous flow)

- ॐ$\sigma=\eta d\varepsilon/dt$; $d\varepsilon/dt = \sigma/\eta$
- ॐor $d\gamma/dt = \tau/\eta$
  - →η = fluid/solid viscosity



Maxwell model



⇒Two element, spring (s) in series with a dashpot (d)

⇒When stress is applied, it is uniform throughout the element

- $\sigma_{applied} = \sigma_s = \sigma_d$

⇒Total strain, $\varepsilon_t = \varepsilon_s + \varepsilon_d$

Maxwell model

$\Rightarrow$Then $\sigma_s = E_s\varepsilon_s = \sigma_d = \eta(d\varepsilon_d /dt)$

$\Rightarrow$$\varepsilon_s = \sigma_s/E$ and $\varepsilon_d = \int \sigma_d/\eta dt$, $\sigma_s=\sigma_d=\sigma$

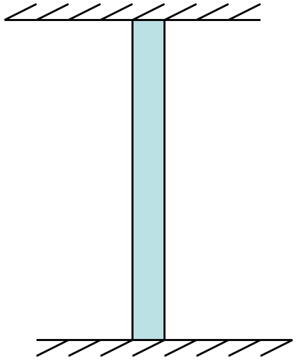
$$\boxed{\varepsilon_T = \frac{\sigma}{E_s} + \int \frac{\sigma}{\eta} dt}$$

$$\boxed{\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}}$$

$$\varepsilon(t) = \frac{\sigma}{E_s} + \frac{\sigma}{\eta} t$$

Characteristics of the Maxwell model

⇒ *For stress relaxation*



- ॐ Sample at σ_o held between 2 fixed plates
  - → Allowed to relax
- ॐ $\varepsilon_t = \varepsilon_o$
  - → Strain rate is kept 0

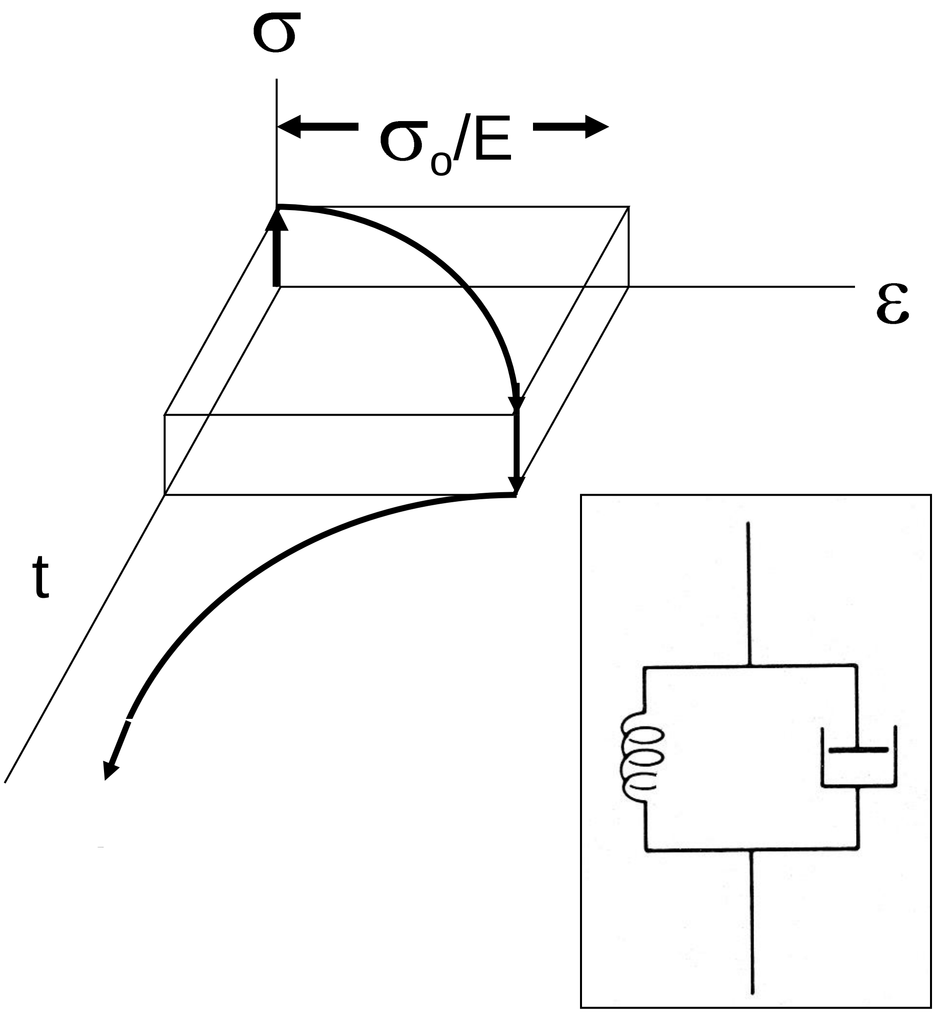
⇒ Then $$\dot{\varepsilon} = 0 = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

$$\sigma(t) = \sigma_o \exp(-Et/\eta) = \sigma_o \exp(-t/\tau)$$

- → Relaxation time (τ) defined as η/E
- → $t >> \tau$, viscous component only
- → $t << \tau$, elastic component only

Stress decreases as a function of increasing time

Voight model



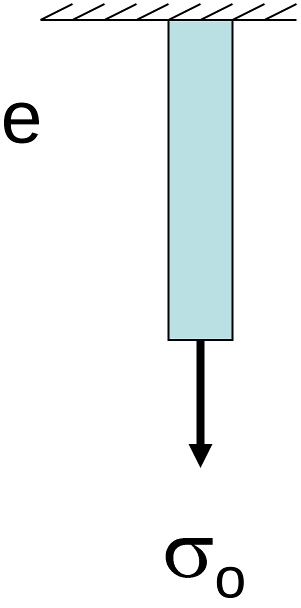
- ⇒Two element system, spring and dashpot in parallel
- ⇒When stress is applied, $\varepsilon_t = \varepsilon_s = \varepsilon_d = \varepsilon$
- ⇒And $\sigma_{applied} = \sigma_s + \sigma_d$
- ⇒Then

$$\sigma_T(t) = E_s\varepsilon + \eta\frac{d\varepsilon}{dt}$$

Characteristics of the Voight model

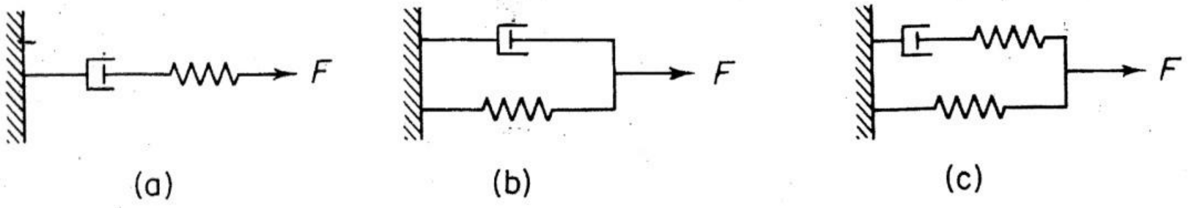
- For creep
  - Load is kept constant on the sample
    - $\sigma(t) = \sigma_o = E\varepsilon + \eta d\varepsilon/dt$
  - Then

$$\varepsilon(t) = \frac{\sigma_o}{E}\left(1 - e^{-t/\tau}\right)$$

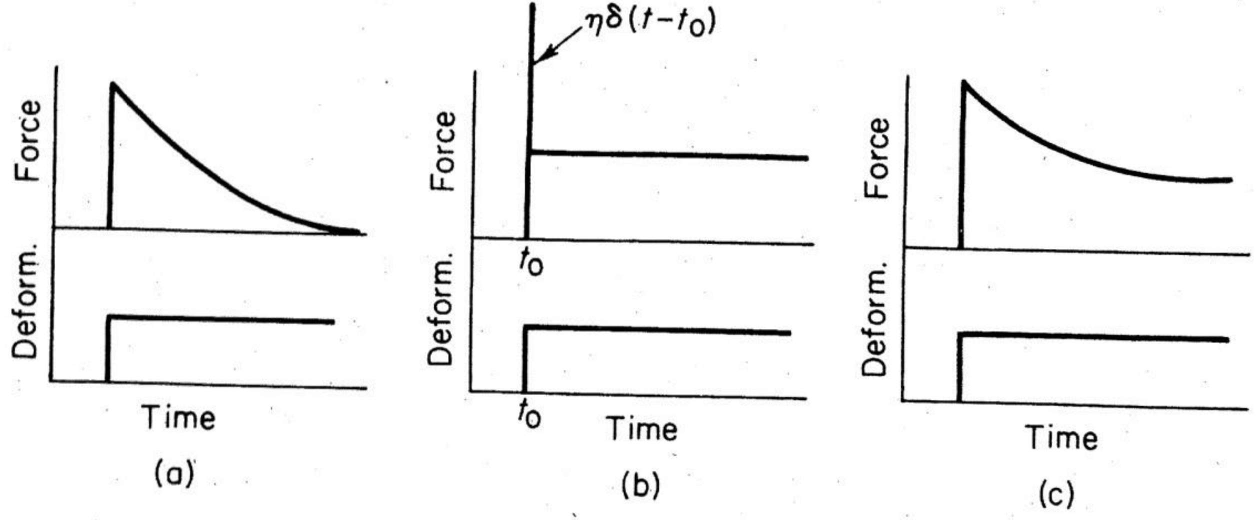


Strain increases to a constant value as a function of increasing time

Stress relaxation, three different models



Models of linear viscoelasticity: (a) Maxwell, (b) Voigt, (c) standard linear solid.

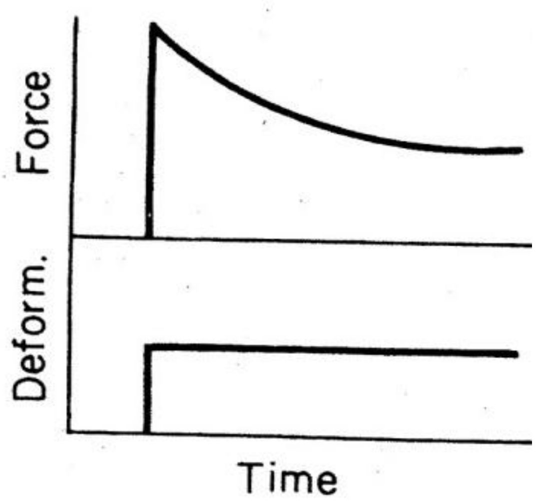


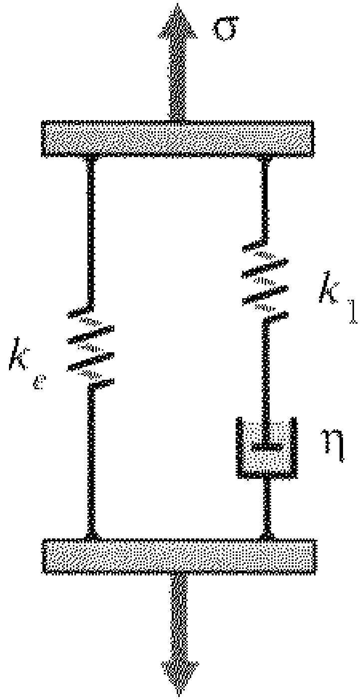
Relaxation functions of (a) Maxwell, (b) Voigt, and (c) standard linear solid.

Stress relaxation, Standard Linear Solid model (three elements model)

In the case of stress relaxation, the strain is a constant ε_0

$$\sigma(t) = \varepsilon_0 \left(k_e + k_1 e^{-\frac{t}{\tau}} \right)$$





Four element model for creep

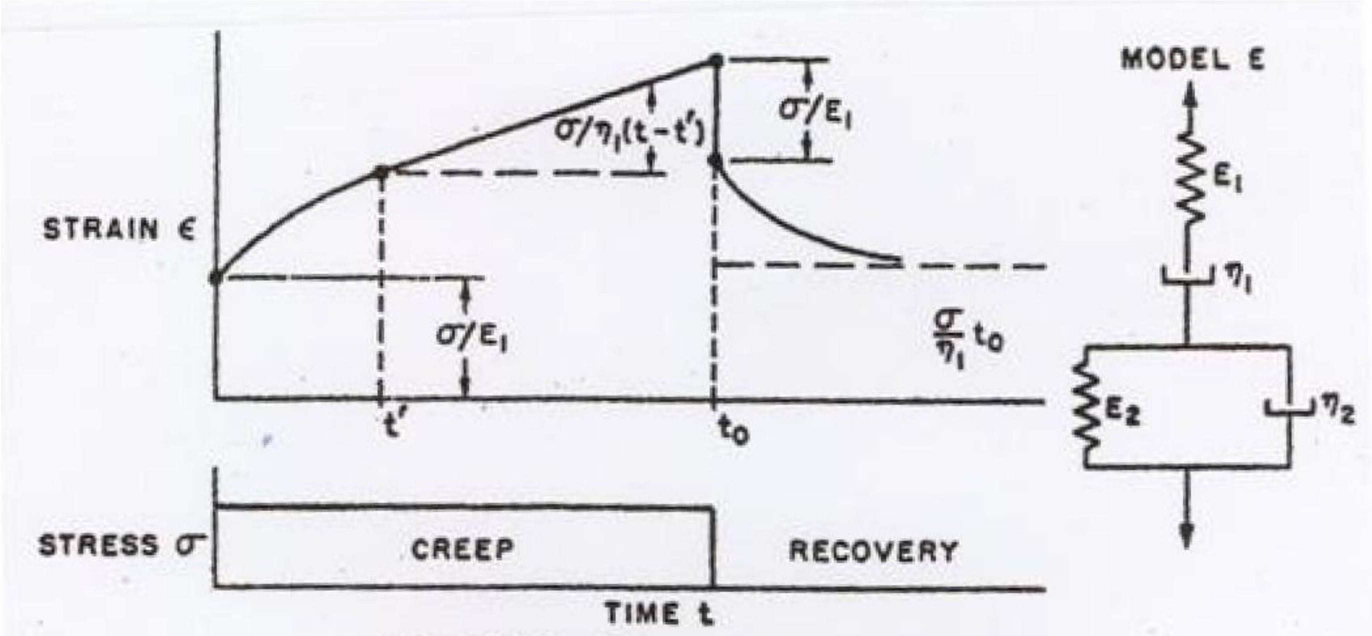


FIG. X6.3 Four Element Model

$$\sigma > 0 : \varepsilon(t) = \frac{\sigma_o}{E_1} + \frac{\sigma_o}{\eta_1} t + \frac{\sigma_o}{E_2}(1 - e^{-t/\tau_2}), \tau_2 = \frac{\eta_2}{E_2};$$

$$\sigma = 0 : \varepsilon(t) = \varepsilon_2(t = t_0) e^{-(t-t_0)/\tau_2} + \frac{\sigma}{\eta_1} t_0.$$

First two terms in the top equation corespond to Maxwell model and third to Voight model

Strain (t) = elastic + viscous + viscoelastic.

We need four equations to estimate 4 parameters in four elements model. For example,

E_1 can be found from the jumps of strain at t=0 and at t=t_0 based on the equation below (these jumps of strains are equal to each other):

$$\varepsilon(t=0) = \sigma/E_1$$

Second equation (to find η_1) can be based on the measured strain limit at t>>t_0:

$$\varepsilon(t >> t_0) = t_0\, \sigma/ \eta_1$$

or it can be based on the measured slop of a straight line at t>>τ_2, and t_0>t>t$'$:

$$\dot{\epsilon}(t_0 > t > t') = \sigma/\eta_1$$

Third equation (to find η_2) can be based on the measured slop at the vicinity of t=0:

$$\dot{\epsilon}(t=0) = \sigma/\eta_1 + \sigma/\eta_2$$

Fourth equation (to estimate E_2, if t>>τ_2) can be based on the measured value of strain at t=t_0 (before the drop of the stress)

$$\varepsilon(t_0) \approx \frac{\sigma}{E_1} + \frac{\sigma}{E_2} + \frac{\sigma}{\eta_1} t_0$$

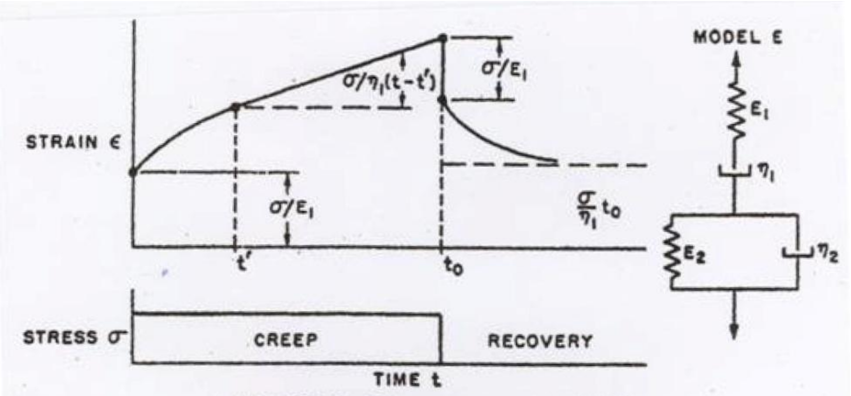
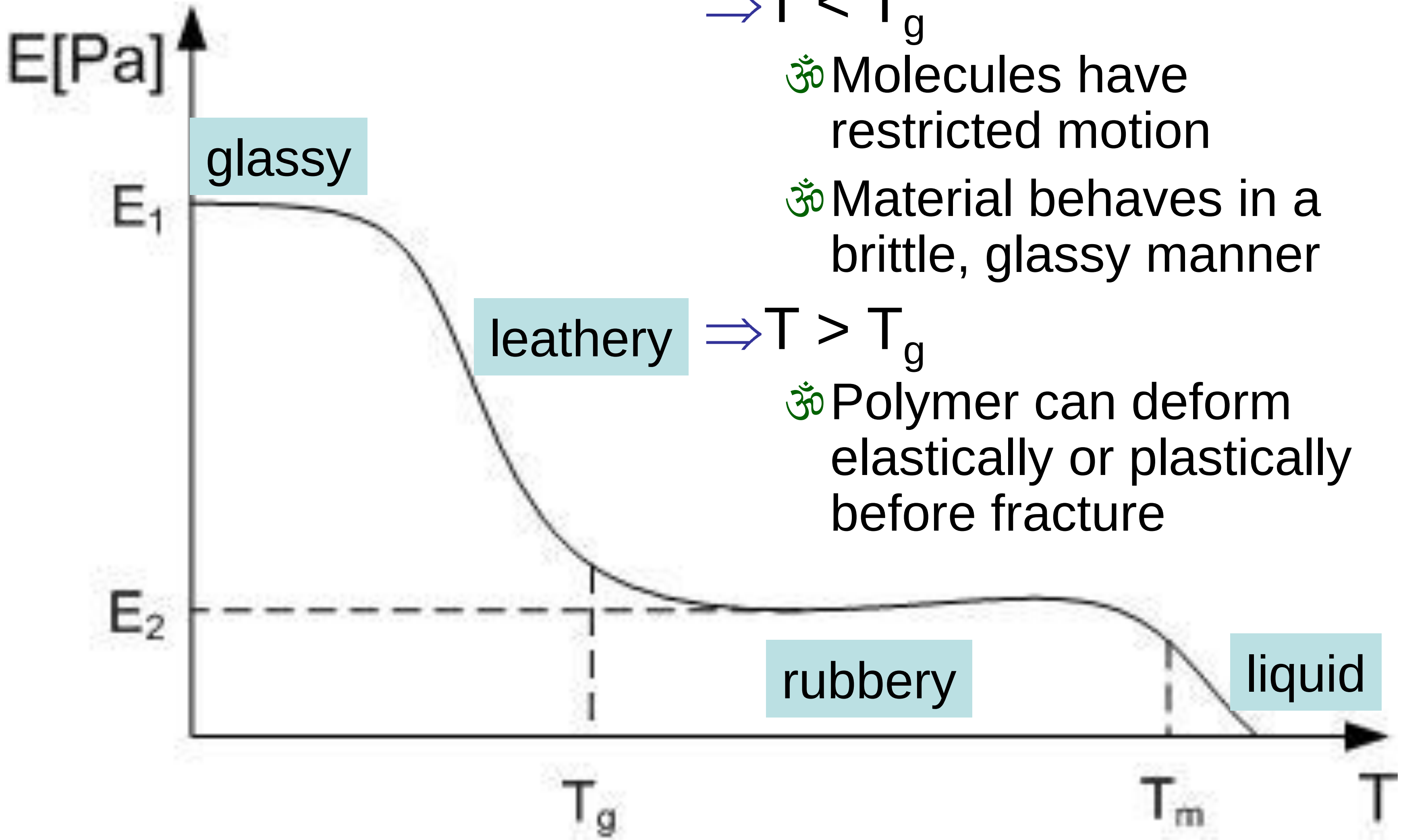
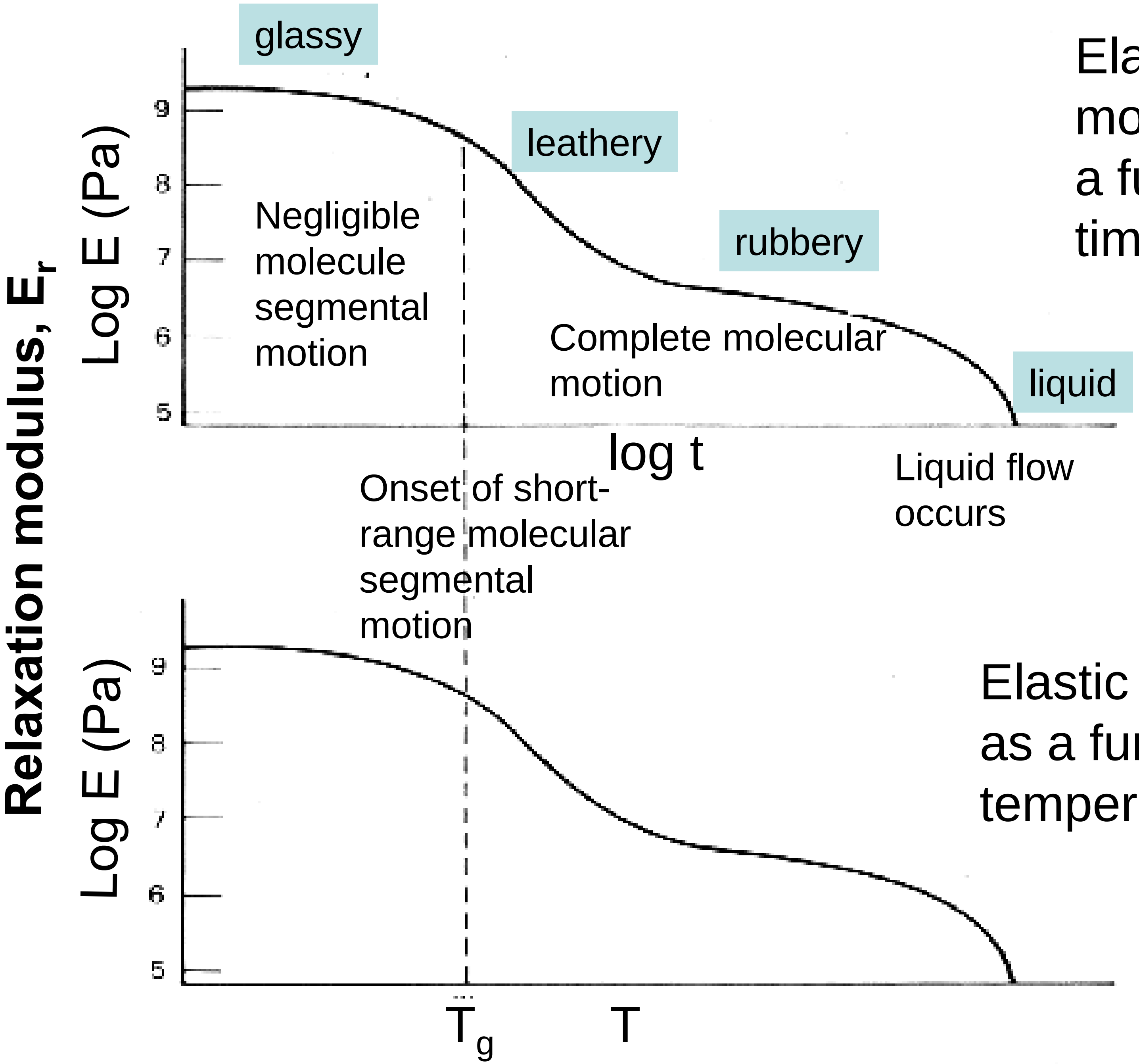


FIG. X6.3 Four Element Model

Glass transition temperature, T_g



- ⇒ $T < T_g$
  - Molecules have restricted motion
  - Material behaves in a brittle, glassy manner
- ⇒ $T > T_g$
  - Polymer can deform elastically or plastically before fracture



Negligible molecule segmental motion

Complete molecular motion

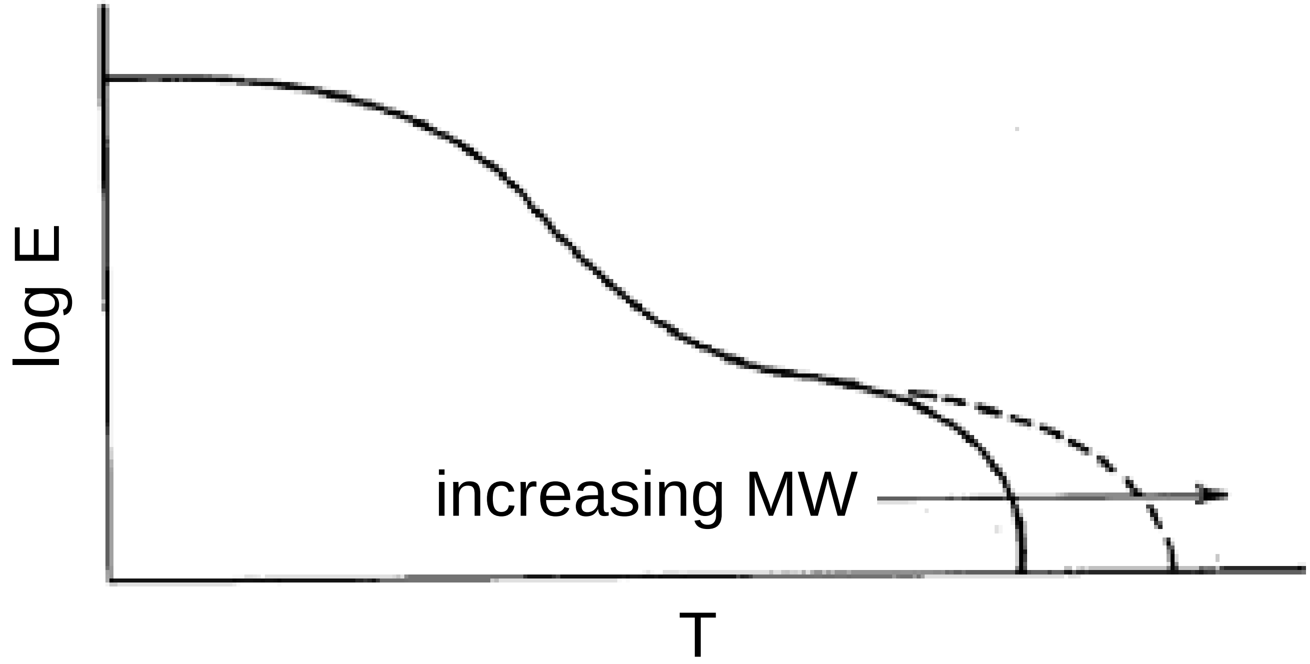
Onset of short-range molecular segmental motion

Liquid flow occurs

Elastic modulus as a function of time

Elastic modulus as a function of temperature

Effect of molecular weight on elastic modulus as a function of temperature



Laboratory objectives

- You are given compact test samples
  - 3 different thermoplastic polymers
  - Different thicknesses
  - Different crack geometries
- You will
  - Calculate K_{Ic}
  - Perform stress relaxation and creep experiments
    - Determine viscosity and relaxation time
    - Plot the creep modulus
      - Determine relaxation times for various materials
  - Identify the effects of temperature on K_{Ic}

ASTM Standards

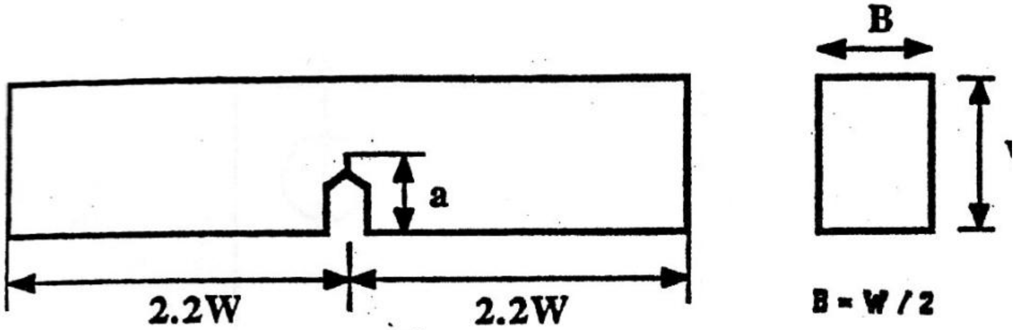
Mechanical tests ***in industrial environment*** must follow ASTM Standards. For example,

1. **ASTM D 5045-99** (reapproved 2007) – Standard Test Methods for Plane –Strain Fracture Toughness and Strain Energy release Rate of Plastic Materials
2. **ASTM D 2990-01** - – Standard Test Methods for tensile, Compressive and Flexural Creep and Creep-Rupture of Plastics

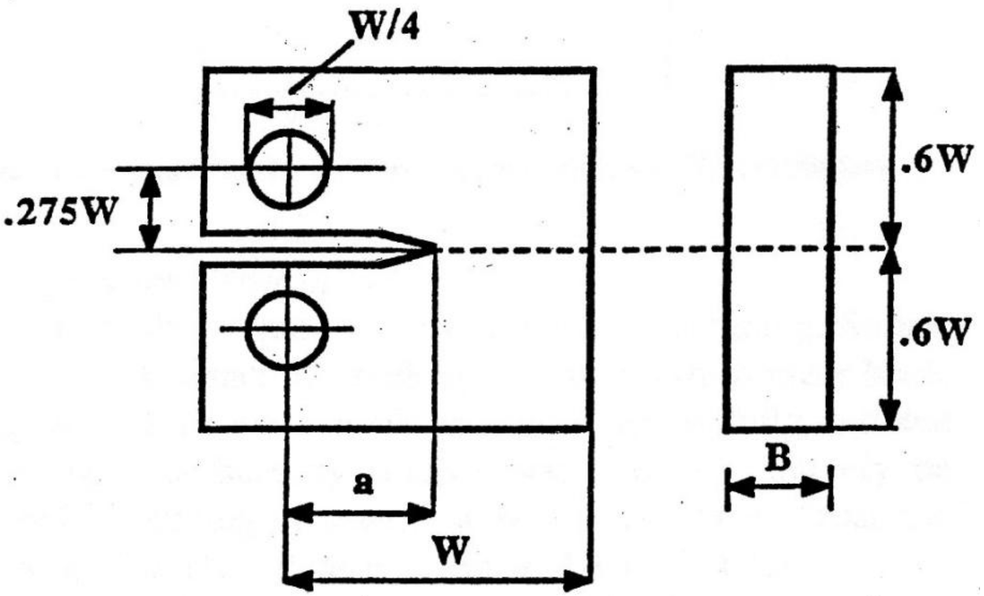
Both Standards are available on the website for 171A.

Requirements for sample sizes

ASTM D 5045-99 – Standard Test Methods for Plane –Strain Fracture Toughness and Strain Energy Release Rate of Plastic Materials



a Three Point Bend Specimen (SENB)



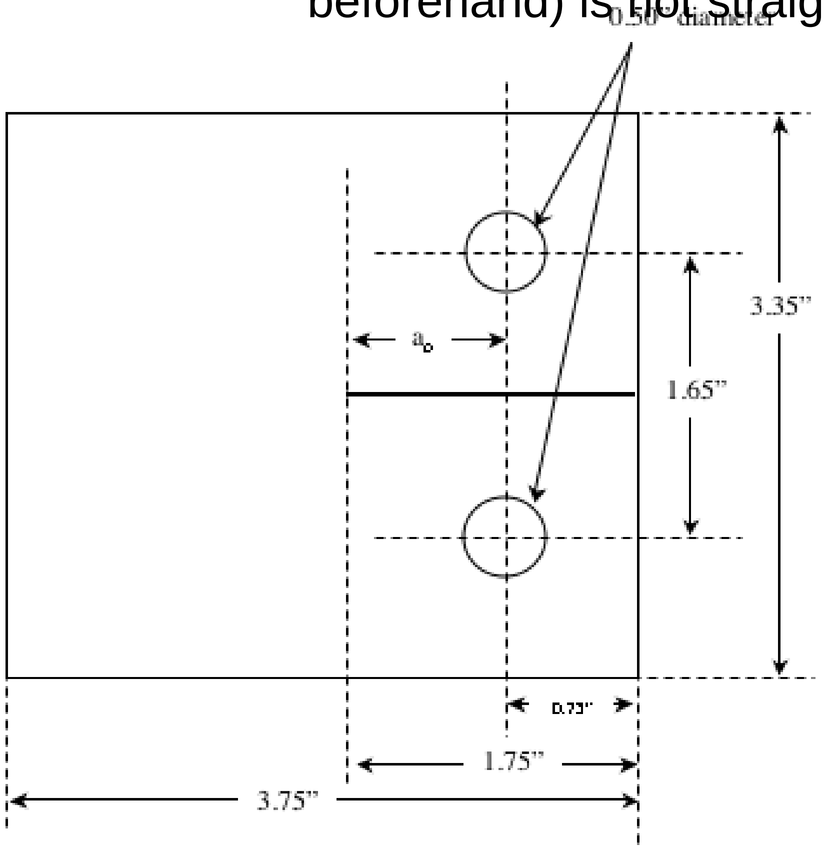
b Compact Tension Configuration (CT)

FIG. 3 Specimen Configuration as in Test Method E 399

Fracture toughness of material reflects its yield strength and its thickness, thus the design of specimens to properly measure fracture toughness (typically not known beforehand) is not straightforward (R.P. Wei, 2010).

Specimen and crack geometry of test specimens

Fracture toughness of material reflects its yield strength and its thickness, thus the design of specimens to properly measure fracture toughness (typically not known beforehand) is not straightforward



$$K = \frac{P}{BW^{1/2}} f\left(\frac{a_0}{W}\right)$$

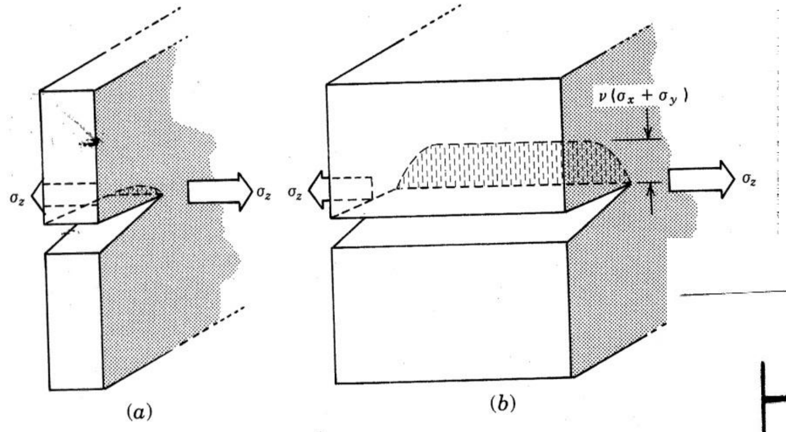
P = applied load
a_0 = crack length
W = specimen width
B = specimen thickness

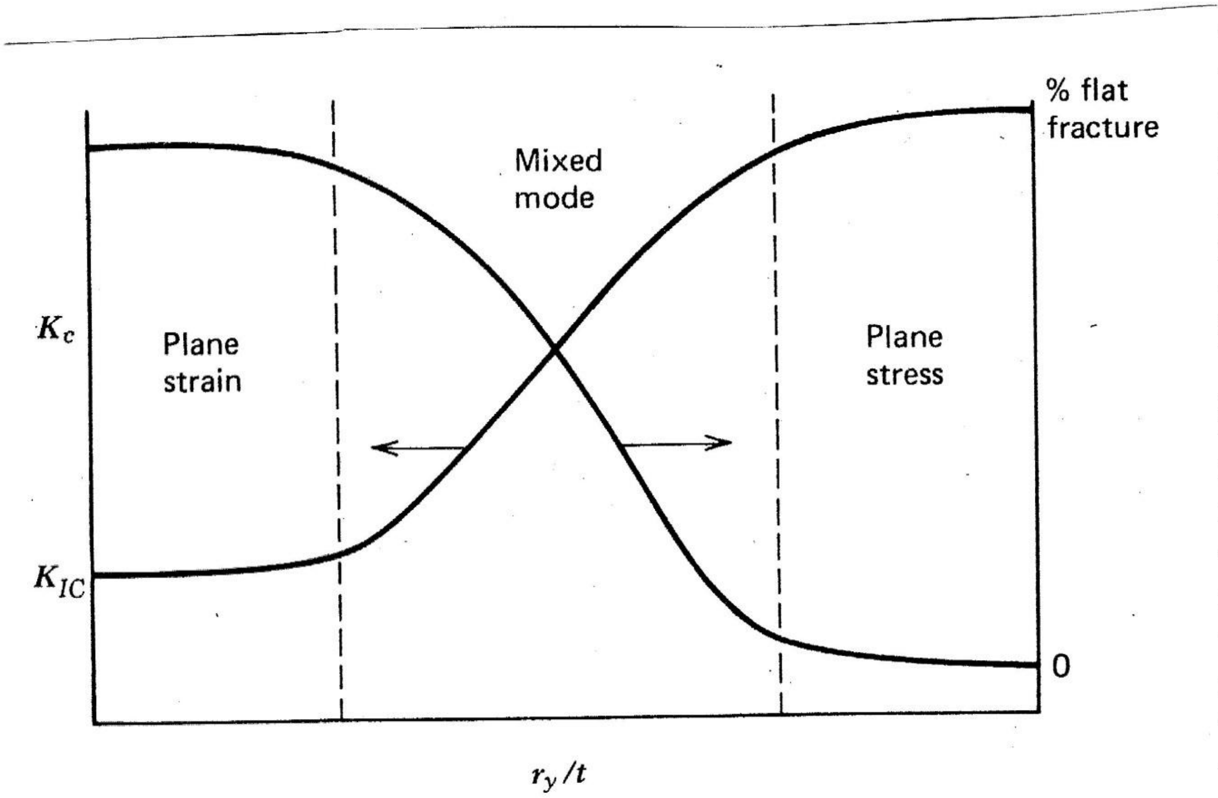
Specimen and crack geometry

$$f\left(\frac{a}{W}\right)=\frac{2+a/W}{(1-a/W)^{3/2}}\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2\right.$$

$$\left.+14.72\left(\frac{a}{W}\right)^3-5.6\left(\frac{a}{W}\right)^4\right]$$

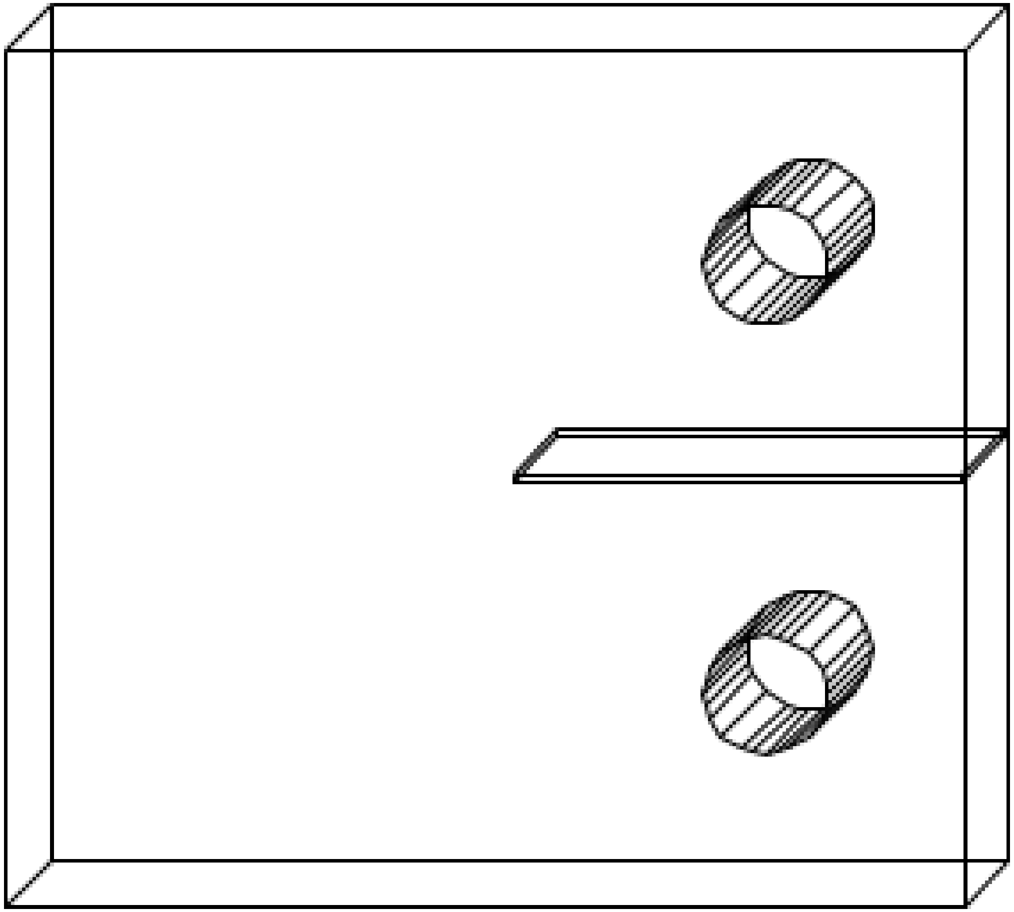
Plane stress (thin sheet, (a)) and plane strain (thick plate, (b)) conditions





Effect of relative plastic zone size to plate thickness on fracture toughness and macroscopic fracture surface appearance. Plane-stress state associated with maximum toughness and slant fracture. Plane-strain state (the plane-strain fracture toughness K_{IC}) associated with minimum toughness and flat fracture (R.W. Hertzberg, 1989, p.297).

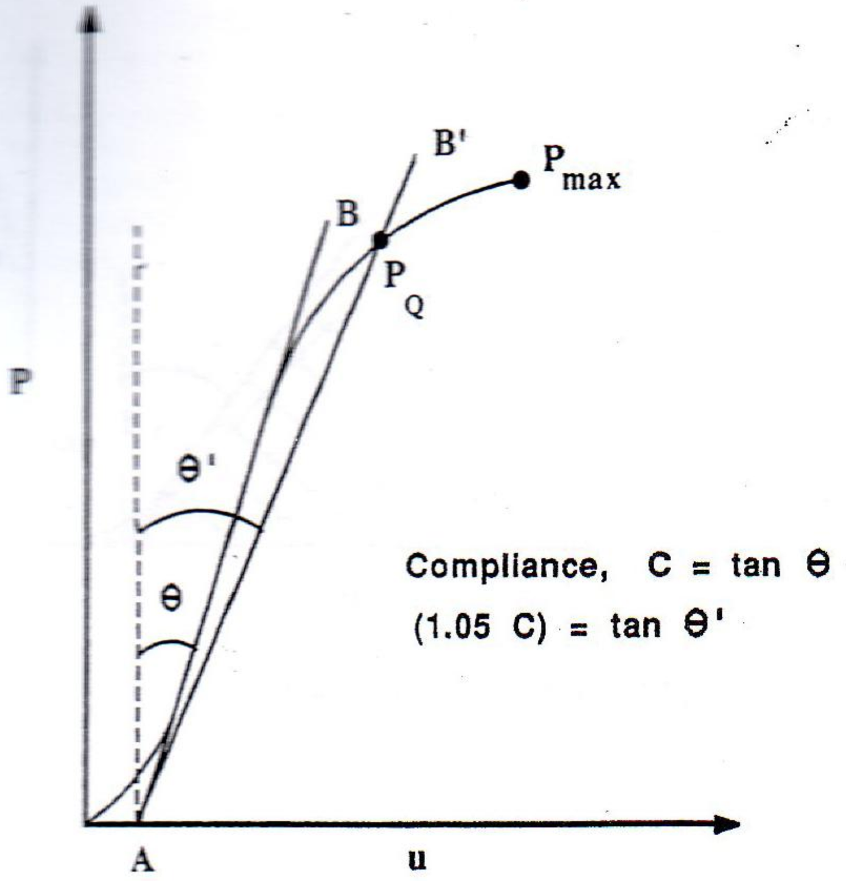
Materials and parameters in Compact Tension Configuration:
Room T (week 1)



Polymer specimen
- polycarbonate
- PMMA
- polypropylene

Specimens
- various thickness
- various crack lengths
- various crack radii
- constant width
- constant length

Interpretation of results



NOTE 1—C is the inverse slope of line AB.

FIG. 5 Determination of C and P_Q

1. Draw a straight line AB
2. Draw a second line AB’ with a compliance 5% greater than that of line AB
3. If maximum load that the specimen was able to sustain, P_{max}, falls within lines AB and AB’ use P_{max} to calculate K_Q
4. If P_{max} falls outside line AB and line AB’ then use the intersection of line AB’ and the load curve as P_Q, furthermore if $P_{max} / P_Q < 1.1$ use P_Q in the calculation of K_Q
5. However if $P_{max} / P_Q > 1.1$ the test is invalid

Did you get a valid value for K_{IC}?

The following size criteria must be satisfied to accept valu of K_{Q} (the conditional or trial K_{IC}) as K_{IC}

B, a, $(W\text{-}a) > 2.5\ (K_{\mathrm{Q}}/\sigma_{\mathrm{y}})^2$,

Where:

σ_{y} is the yield stress of the material in the condition of testing. *Otherwise the test is not a valid K_{IC} test*.

The criteria require that *B* must be sufficient to ensure plane strain and that (*W*-*a*) be sufficient to avoid excessive plasticity in the ligament

How to use crack-tip radius and crack length data

⇒From the work to fracture

$$\sigma_c = \sqrt{E\gamma_s / a_o}$$

⇒And the stress intensity equation

$$\sigma_{\max} = 2\sigma_{appl}\sqrt{a / \rho}$$

⇒Assume that $\sigma_c = \sigma_{max}$

⇒Then

$$\sigma_{appl} = \sqrt{\frac{E\gamma / a_o}{4(a / \rho)}} = \sqrt{\frac{E\gamma}{4a_o}}\sqrt{\frac{\rho}{a}} = A\left(\sqrt{\frac{\rho}{a}}\right)$$

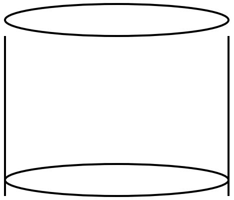
measured

calculate
for known ρ,c

measured

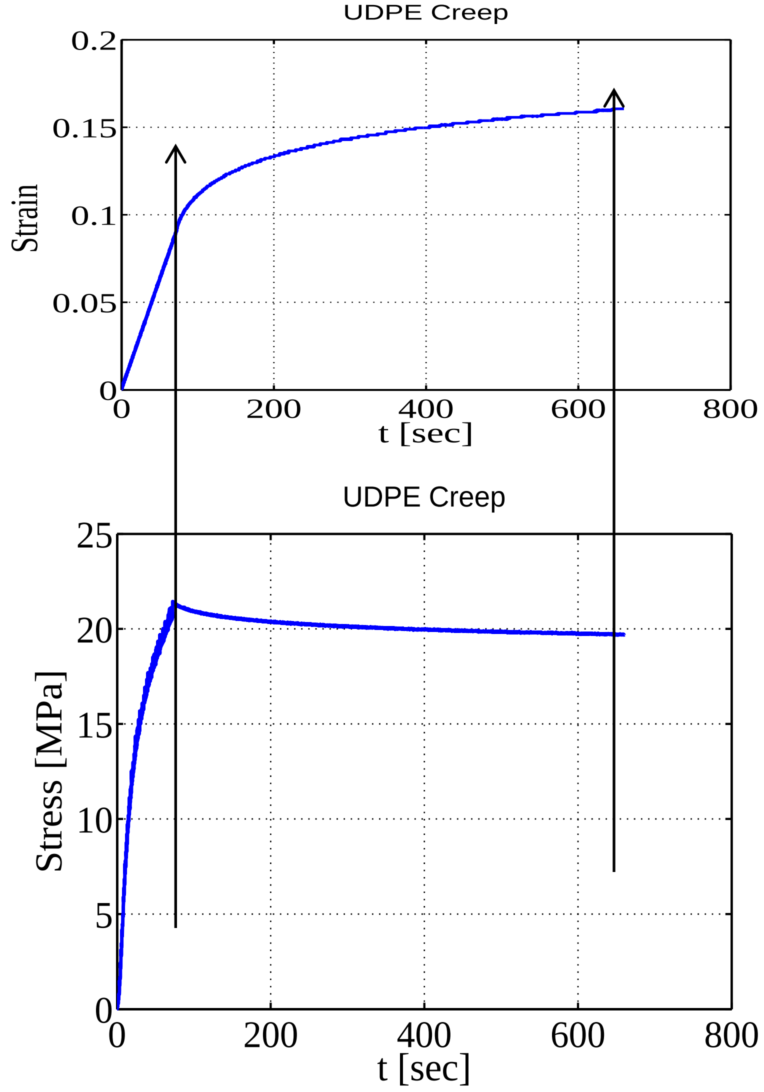
Deformation of polymers (week 2)

- Samples will be tested in compression
  - Barrel shaped configuration



- In one case, the platens are fixed
  - Stress relaxation, Maxwell model
    - Stress is recorded, strain constant
- In the other case, the measured load is fixed
  - Creep, Voight model or better Four element model
    - Strain is recorded, stress constant

UHDP (Ultra-high density polyethylene) Creep



Experimental data

Model behavior under suddenly applied load

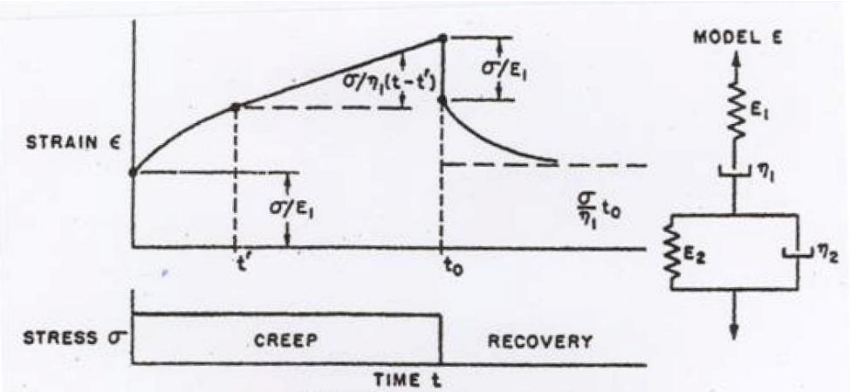
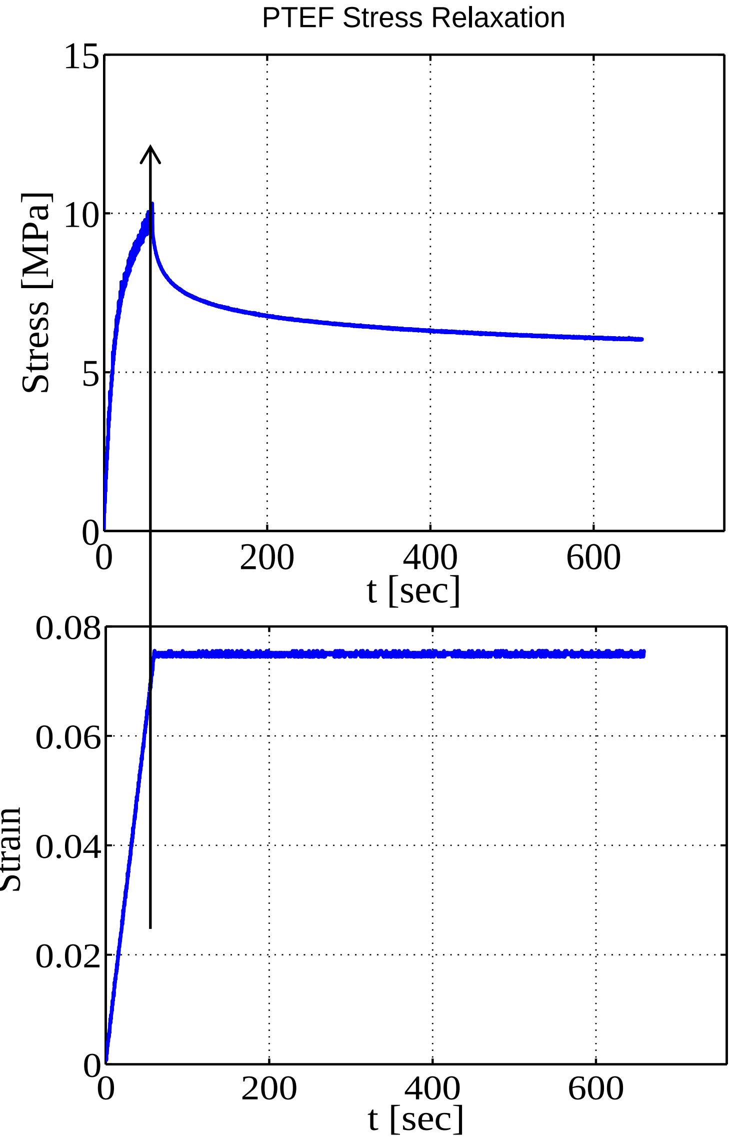


FIG. X6.3 Four Element Model

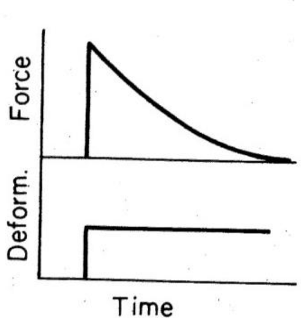
(1) Model can be applied to interpret experimental data starting from the point (see vertical arrow) where machine applied constant load (approximately at *t*=70 sec, for convenience in the graph the sign of load was changed); (2) After load was removed (approximately at *t*=650 sec, see another vertical arrow) strains should be calculated based on manual measurements of sample size.

PTFE Stress Relaxation

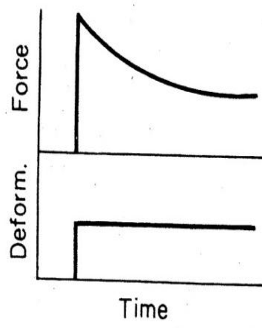


1.The test started when strain reach the constant value, (see vertical arrow) (in this example approximately at *t*=70 sec). For convenience in the graphs the signs of load and strain were changed);

2. Based on the behavior of your sample you need to decide which model to select to interpret the data: Maxwell model, (if final stress is assumed to relax to zero)



or standard liner solid model (three elements or Kelvin model) if the stress assumed a constant value



Temperature effects on K_{Ic} (week 3)

⇒You will use the data from the first week (20˚C)

⇒You will perform the same measurements while the samples are in an ice bath (0˚C)

⇒You will perform the same measurements while the samples are heated by a heat lamp