
Linear Circuit Experiment (MAE171a)

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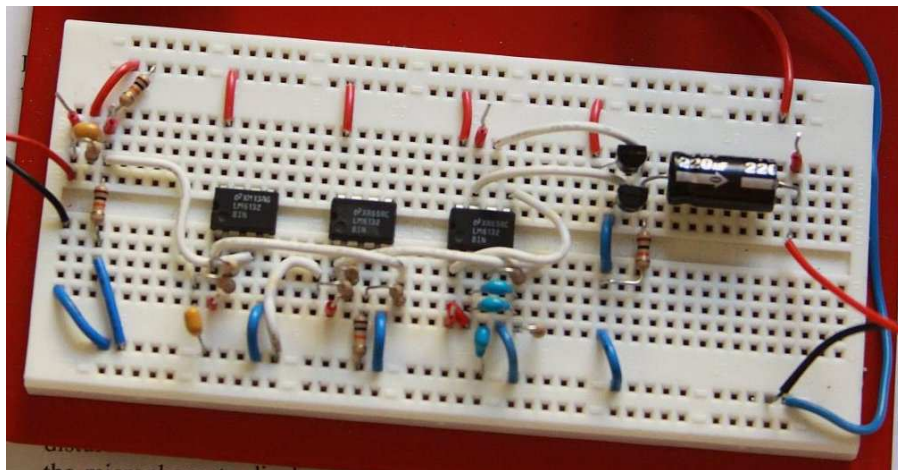
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class information and lab handouts will be available on
<http://maecourses.ucsd.edu/labcourse/>

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Main Objectives of Laboratory Experiment:

modeling, building and debugging of op-amp based linear
circuits for standard signal conditioning



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Main Objectives of Laboratory Experiment:

modeling, building and debugging of op-amp based linear circuits for standard signal conditioning

Ingredients:

- modeling of standard op-amp circuits
- signal conditioning with application to audio (condensor microphone as input, speaker as output)
- implementation & verification of op-amp circuits
- sensitivity and error analysis

Background Theory:

- Operational Amplifiers (op-amps)
- Linear circuit theory (resistor, capacitors)
- Ordinary Differential Equations (dynamic analysis)
- Amplification, differential & summing amplifier and filtering

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Outline of this lecture

- Linear circuits & purpose of lab experiment
- Background theory
 - op-amp
 - linear amplification
 - single power source
 - differential amplifier
 - summing circuit
 - filtering
 - simulation
- Laboratory work
 - week 1: microphone and amplification
 - week 2: audio amplification and mixing
 - week 3: filtering and complete circuit (with power boost)
- Summary
- What should be in your report

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Linear Circuits & Signal Conditioning

Signal conditioning crucial for proper signal processing.

Applications may include:

- **Analog to Digital Conversion**
 - Resolution determined by number of bits of AD converter
 - Amplify signal to maximum range for full resolution
- **Noise reduction**
 - Amplify signal to allow processing
 - Filter signal to reduce undesired aspects
- **Feedback control**
 - Feedback uses reference $r(t)$ and measurement $y(t)$
 - Compute difference $e(t) = r(t) - y(t)$
 - Amplify, Integrate and or Differentiate $e(t)$ (PID control)
- **Signal generation**
 - Create sinewave of proper frequency as carrier
 - Create blockwave of proper frequency for counter
 - etc. etc.

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Purpose of Lab Experiment

In this laboratory experiment we focus on a (relatively simple) signal conditioning algorithms: *amplification*, *adding/difference* and basic (at most 2nd order) *filtering*.

Objective: to *model, build and debug op-amp based linear circuits that allow signal conditioning algorithms*.

We apply this to an *audio application*, where the signal of a condenser microphone needs to *amplified, mixed* and *filtered*.

Challenge: single source power supply of 5 Volt. *Avoid clipping/distortion of amplified, mixed and filtered signal*.

Aim of the experiment:

- insight in op-amp based linear circuits (MAE140, MAE170)
- build and debug (frustrating)
- compare theory (ideal op-amp) with practice (build and test)
- verify circuit behavior (experiment and simulation)

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Background Theory - op-amp

op-amp = operational amplifier

more precise definition:

DC-coupled high-gain electronic voltage amplifier with differential inputs and a single output.

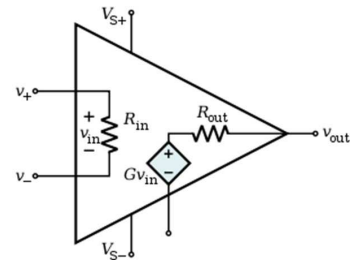
- **DC-coupled**: constant (Direct Current) voltage at inputs results in a constant (DC) voltage at output
- **differential inputs**: two inputs V_- and V_+ and the difference $V_\delta = V_+ - V_-$ is only relevant and amplified
- **high-gain**: $V_{out} = G(V_+ - V_-)$ where $G \gg 1$.

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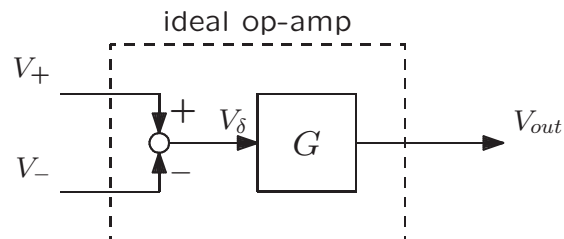
Background Theory - op-amp

Ideal op-amp (equivalent circuit right):

- input impedance: $R_{in} = \infty \Rightarrow i_{in} = 0$
- output impedance: $R_{out} = 0$
- gain: $V_\delta = (V_+ - V_-)$, $V_{out} = GV_\delta$, $G = \infty$
- rail-to-rail: $V_{S-} \leq V_{out} \leq V_{S+}$



Ideal op-amp (block diagram below)

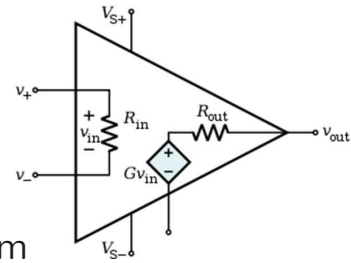


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Background Theory - op-amp

Infinite input impedance ($R_{in} = \infty$) useful to minimize load on sensor/input.

Zero output impedance ($R_{out} = 0$) useful to minimize load dependency and obtain maximum output power.



Rail-to-rail operation to maximize range of output V_{out} between negative source supply V_{S-} and positive source supply V_{S+} .

But why (always) infinite gain G ? Obviously:

$$V_{out} = \begin{cases} V_{S+} & \text{if } V_+ > V_- \\ 0 & \text{if } V_+ = V_- \\ V_{S-} & \text{if } V_+ < V_- \end{cases}$$

does not very useful with any (small) noise on V_+ or V_- .

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Background Theory - op-amp

Usefulness of op-amp with high gain G only by **feedback!**

Consider open-loop behavior:

$$V_{out} = GV_{\delta}, \text{ where } V_{\delta} = V_+ - V_-$$

and create a feedback of V_{out} by choosing

$$V_- = KV_{out}$$

to make

$$V_{\delta} = V_+ - KV_{out}$$

Then

$$V_{out} = GV_{\delta} = GV_+ - GKV_{out}$$

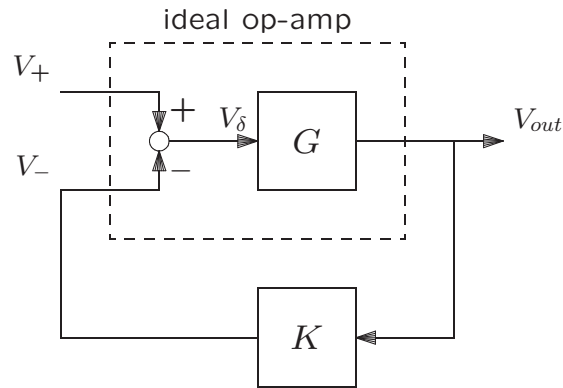
allowing us to write

$$V_{out} = \frac{G}{1 + GK}V_+$$

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Background Theory - op-amp

So, with the feedback $V_- = KV_{out}$ we obtain $V_{out} = \frac{G}{1 + GK} V_+$



In case $G \rightarrow \infty$ we get well-defined relation:

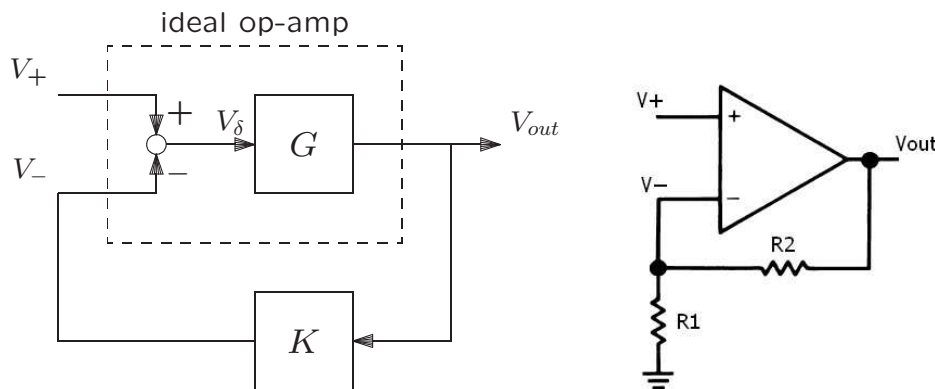
$$V_{out} = \frac{1}{K} V_+$$

- Don't care what gain G is, as long as it is LARGE
- Make sure K is well-defined and accurate
- If $0 < K < 1$ then V_+ is nicely amplified to V_{out} by $1/K$

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Background Theory - op-amp

Amplification $1/K$ by feedback K of ideal high gain op-amp:



Series of R_1 and R_2 leads to voltage divider on V_- given by:

$$V_- = \frac{R_1}{R_1 + R_2} \cdot V_{out} = K \cdot V_{out}, \quad 0 < K \leq 1$$

and with ideal high gain op-amp we get

$$\lim_{G \rightarrow \infty} V_{out} = \lim_{G \rightarrow \infty} \frac{G}{1 + GK} V_+ = \frac{1}{K} V_+ = \frac{R_1 + R_2}{R_1} V_+ = \left(1 + \frac{R_2}{R_1} \right) V_+$$

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Background Theory - non-inverting amplifier (voltage follower)

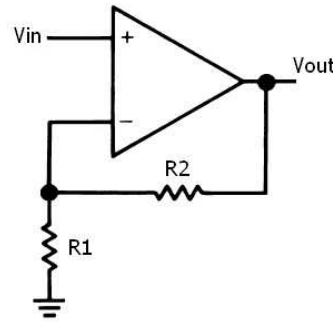
Our first application circuitry:

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

So-called **voltage follower** in case

$$R_1 = \infty \text{ (not present) and } R_2 = 0$$

where $V_{out} = V_{in}$ but **improved output impedance!**



Quick (alternative) analysis based on $V_+ = V_-$ and $i_+ = i_- = 0$:

- Since $i_- = 0$ and series R_1 , R_2 we have $V_- = \frac{R_1}{R_1 + R_2} V_{out}$
- Hence

$$V_{in} = V_+ = \frac{R_1}{R_1 + R_2} V_{out} \Rightarrow V_{out} = \frac{R_1 + R_2}{R_1} V_{in} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

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Background Theory - inverting amplifier

Similar circuit but now negative sign:

$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

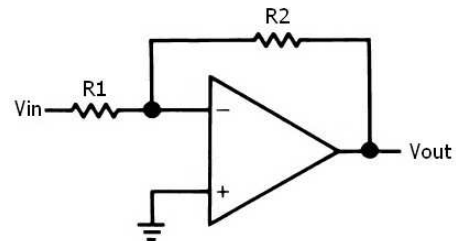
Quick (alternative) analysis based on $V_+ = V_-$ and $i_+ = i_- = 0$:

- With $V_- = V_+ = 0$ and $i_- = 0$, Kirchoff's Current Law indicates

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

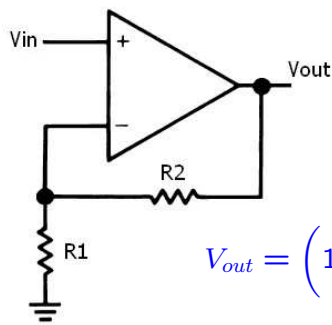
- Hence

$$\frac{V_{out}}{R_2} = -\frac{V_{in}}{R_1} \Rightarrow V_{out} = -\frac{R_2}{R_1} V_{in}$$

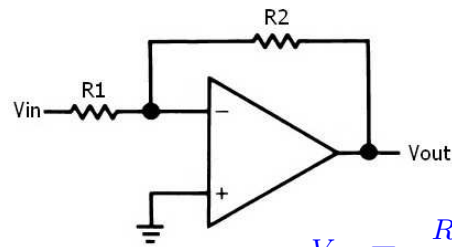


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Background Theory - effect or rail (source) voltages



$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$



$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

Formulae are for ideal op-amp with **boundaries imposed by negative source supply V_{S-} and positive source supply V_{S+}**

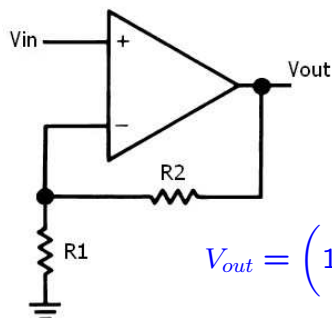
$$V_{S-} \leq V_{out} \leq V_{S+} \quad (\text{rail-to-rail op-amp})$$

Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground):

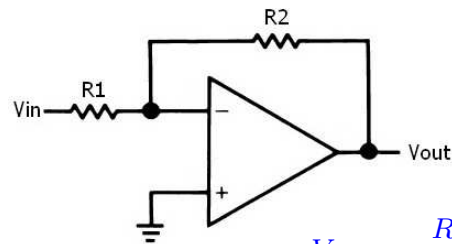
- Limits use of inverting amplifier ($V_{out} < 0$ not possible)
- Limits use of large gain R_2/R_1 ($V_{out} > V_{cc}$ not possible)

Design challenge: $0 < V_{out} < V_{cc}$ to **avoid 'clipping' of V_{out}** .

Background Theory - effect or rail (source) voltages



$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$



$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground) complicates amplification of

$$V_{in}(t) = a \sin(2\pi ft)$$

as $-a < V_{in}(t) < a$ (both positive and negative w.r.t. ground).

Example: audio application (as in our experiment).

To ensure $0 < V_{out} < V_{cc}$ provide **offset compensation**

$$V_{in}(t) = a \sin(2\pi ft) + a$$

to ensure $V_{in}(t) > 0$ and use **non-inverting amplifier**.

Background Theory - differential amplifier

Instead of amplifying one signal, amplify the difference:

$$V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$$

Difference or differential amplifier is found by inverting amplifier and adding signal to V_+ via series connection of R_3 and R_4 . Analysis:

- With $i_+ = 0$ the series of R_3 and R_4 leads to $V_+ = \frac{R_4}{R_3 + R_4} V_2$
- With $V_- = V_+$ and Kirchhoff's Current Law we have

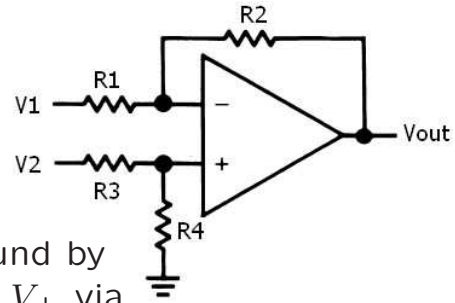
$$\frac{V_1 - \frac{R_4}{R_3 + R_4} V_2}{R_1} + \frac{V_{out} - \frac{R_4}{R_3 + R_4} V_2}{R_2} = 0$$

Hence

$$V_{out} = \frac{R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} V_2 + \frac{R_4}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1$$

or

$$V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$$



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Background Theory - differential amplifier

Choice $R_1 = R_3$ and $R_2 = R_4$ reduces

$$V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$$

to

$$V_{out} = \frac{R_2}{R_1} (V_2 - V_1)$$

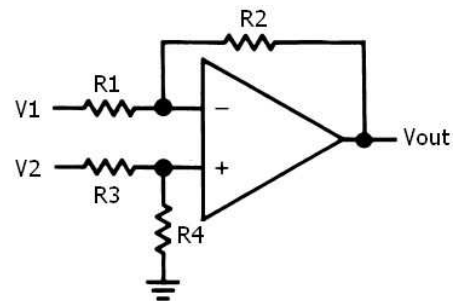
create a difference/differential amplifier.

Further choice of $R_1 = R_3$, $R_2 = R_4$ and $R_2 = R_1$ yields

$$V_{out} = V_2 - V_1$$

and computes the difference between input voltages V_1 and V_2 .

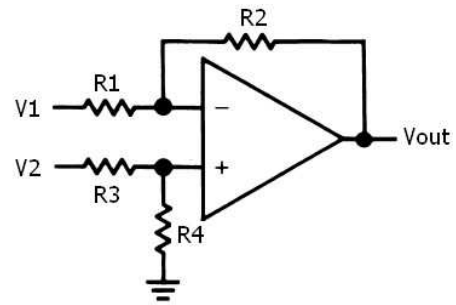
NOTE: $V_2 > V_1$ for a **single voltage power supply** with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground) to avoid clipping of V_{out} against ground.



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Background Theory - more advanced differential amplifiers

Difference amplifier does not have high input impedance (loading of sensors).
Better design with voltage followers:



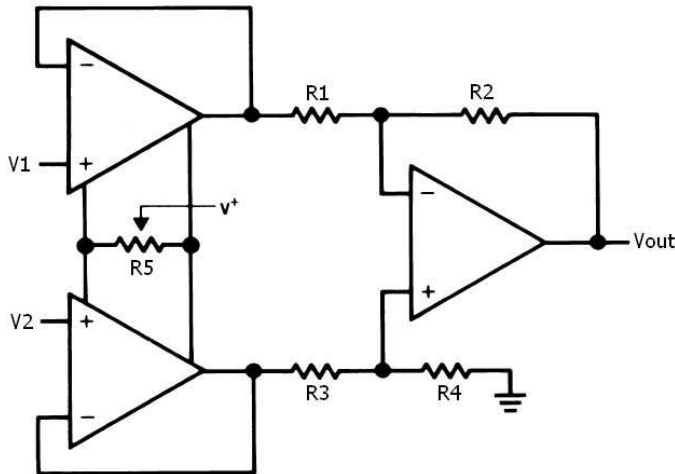
With

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

we have

$$V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

R_5 is used to adjust offset (balance)



Background Theory - more advanced differential amplifiers

Even better differential amplifier that has a **variable gain** is a so-called **instrumentation amplifier**:

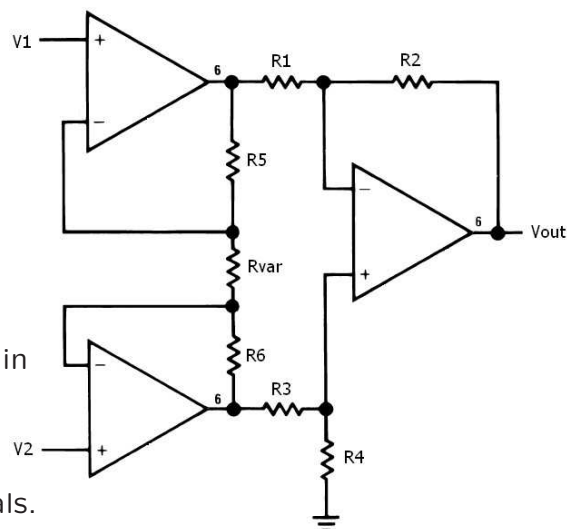
Setting all resistors

$$R_i = R, \quad i = 1, 2, \dots, 5$$

except R_{var} , makes

$$V_{out} = \left(1 + \frac{2R}{R_{var}}\right)(V_2 - V_1)$$

High input impedance and variable gain via an (external) resistor R_{var} makes this ideal for the amplification of (non-grounded) instrumentation signals.



Instrumentation amplifiers are made & sold as a single chip.

Background Theory - inverting summing amplifier

Inverting amplifier can also be extended to add signals:

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

Analysis follows from Kirchhoff's Current Law for the $-$ input of the op-amp:

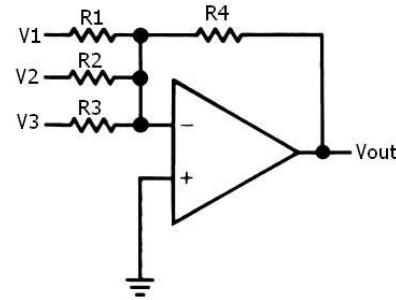
- With $V_- = V_+$ we have $V_- = 0$
- With $i_- = 0$ we have

$$\frac{V_{out}}{R_4} + \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0$$

Hence

$$V_{out} = -R_4 \cdot \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

creating a weighted sum of signals.



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Background Theory - inverting summing amplifier

The choice $R_1 = R_2 = R_3$ reduces

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

to

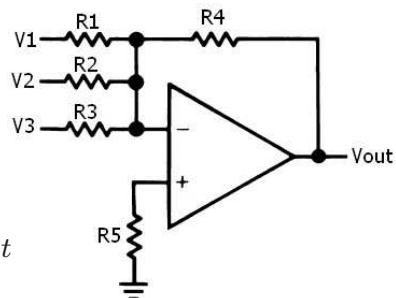
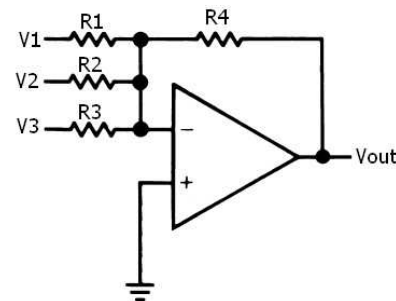
$$V_{out} = -\frac{R_4}{R_1} (V_1 + V_2 + V_3)$$

simply amplifying the sum of the signals.

Oftentimes **extra resistor R_5** is added:

$$\frac{1}{R_5} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

to account for possible small (bias) input currents $i_- \neq 0$, $i_+ \neq 0$. This ensures V_{out} remains sum, without bias/offset.



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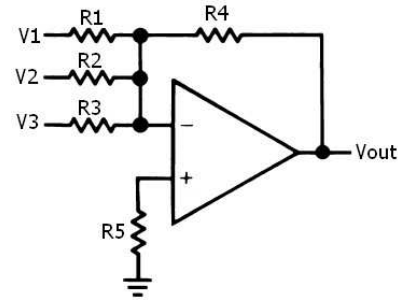
Background Theory - inverting summing amplifier

Inverting summing amplifier:

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

and for $R_1 = R_2 = R_3$:

$$V_{out} = -\frac{R_4}{R_1} (V_1 + V_2 + V_3)$$



has a limitation for single source voltage supplies:

Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$:

- Limits use of inverting summer ($V_{out} < 0$ not possible)
- Limits use of large gain R_4/R_1 ($V_{out} > V_{cc}$ not possible)

'clipping' of V_{out} will occur if sum of input signals is positive.

Background Theory - non-inverting summing amplifier

Based on a non-inverting amplifier signals can also be summed:

$$V_{out} = \left(1 + \frac{R_4}{R_3} \right) \frac{R_1 R_2}{R_1 + R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

Analysis:

- due to $i_- = 0$ we have $V_- = \frac{R_3}{R_3 + R_4} V_{out}$
- Due to $V_+ = V_-$ and $i_+ = 0$ with Kirchoff's Current Law:

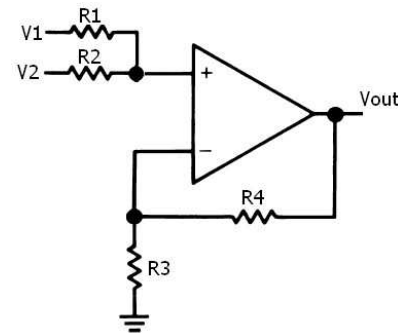
$$\frac{V_1 - \frac{R_3}{R_3 + R_4} V_{out}}{R_1} + \frac{V_2 - \frac{R_3}{R_3 + R_4} V_{out}}{R_2} = 0$$

Hence

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \frac{R_3}{R_3 + R_4} V_{out}$$

and

$$V_{out} = \left(1 + \frac{R_4}{R_3} \right) \frac{R_1 R_2}{R_1 + R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$



Background Theory - non-inverting summing amplifier

The choice $R_1 = R_2$ reduces

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right) \frac{R_1 R_2}{R_1 + R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

to

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right) \frac{V_1 + V_2}{2}$$

indicating amplification of the sum V_1 and V_2 if $R_4 \geq R_3$.

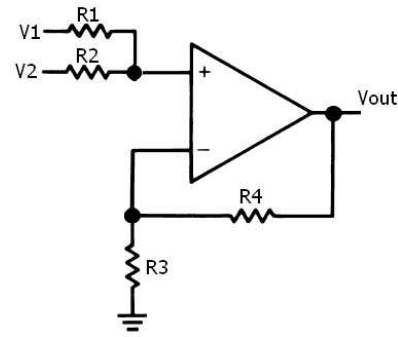
Further choice of also $R_1 = R_2 = R_3 = R_4$ leads to

$$V_{out} = V_1 + V_2$$

indicating a simple summation of V_1 and V_2 .

Unlike inverting summing amplifier, no extra resistor can be added to compensate for bias input current.

Not desirable: source impedance part of gain calculation...



Background Theory - filtering

So far, all circuits were build using op-amps and resistors.

When building filters, mostly **capacitors** are used as negative, positive or grounding elements.

Interesting phenomena: **resistor value of capacitor depends on frequency of signal.**

Analysis for capacitor: **capacitance C is ratio between charge Q and applied voltage V :**

$$C = \frac{Q}{V}$$

Since charge $Q(t)$ at any time is found by flow of electrons:

$$Q(t) = \int_{\tau=0}^t i(\tau) d\tau$$

we have

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C} \int_{\tau=0}^t i(\tau) d\tau$$

Background Theory - filtering

Application of **Laplace transform** to

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C} \int_{\tau=0}^t i(\tau) d\tau$$

yields

$$V(s) = \frac{1}{Cs} i(s)$$

Hence we can define the **impedance/resistance** of a capacitor as

$$R(s) = \frac{V(s)}{i(s)} = \frac{1}{Cs}$$

With Fourier analysis we use $s = j\omega$ and we find the frequency dependent 'equivalent resistor value of a capacitor':

$$R(j\omega) = \frac{1}{jC\omega}$$

This value will allow analysis of op-amp circuits based on resistors (as we have done so far)

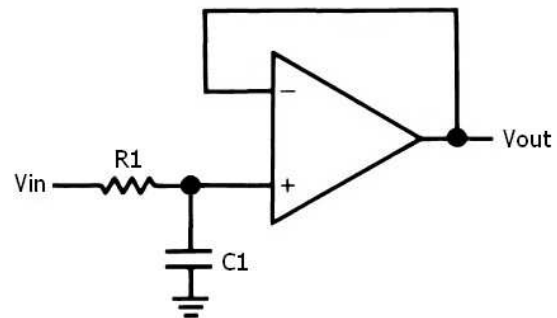
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Background Theory - filter (1st order RC)

Consider simple 1st order RC-filter with voltage follower op-amp.

Due to $i_+ = 0$ we have

$$V_+(j\omega) = \frac{\frac{1}{C_1 j\omega}}{R_1 + \frac{1}{C_1 j\omega}} V_{in}(j\omega)$$



With $V_{out} = V_- = V_+$ we have

$$V_{out}(j\omega) = \frac{1}{R_1 C_1 j\omega + 1} V_{in}(j\omega)$$

This is a 1st order low-pass filter with a cut-off frequency

$$\omega_c = \frac{1}{R_1 C_1} \text{ rad/s or } f_c = \frac{1}{2\pi R_1 C_1} \text{ Hz}$$

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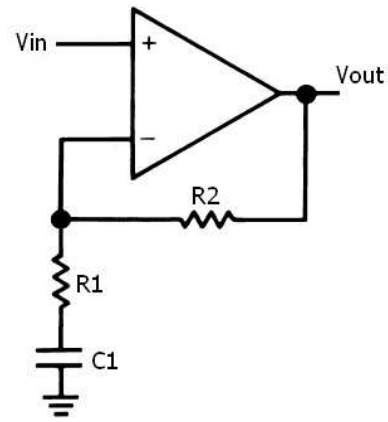
Background Theory - filter (audio amplifier)

Consider circuit of non-inverting amplifier where R_1 is now series of R_1 and C_1 . Equivalent series resistance is given by

$$R_1 + \frac{1}{jC_1\omega}$$

Application of gain formula for non-inverting amplifier yields:

$$V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1 + \frac{1}{jC_1\omega}} \right) V_{in}(j\omega)$$



We can directly see:

- For low frequencies $\omega \rightarrow 0$ we obtain a **Voltage follower** with $V_{out} = V_{in}$
- For high frequencies $\omega \rightarrow \infty$ we obtain our usual **non-inverting amplifier** $V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1} \right) V_{in}(j\omega)$

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Background Theory - filter (audio amplifier)

This transition between Voltage follower to amplifier as a function of the frequency is also called **audio amplifier**.

The transition between low and high frequency can be studied better by writing $V_{out}(s) = G(s)V_{in}(s)$ where $G(s)$ is a **transfer function**.

This allows us to write

$$V_{out}(s) = \left(1 + \frac{R_2}{R_1 + \frac{1}{C_1s}} \right) V_{in}(s)$$

as

$$V_{out}(s) = \left(1 + \frac{R_2C_1s}{R_1C_1s + 1} \right) V_{in}(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1} V_{in}(s)$$

making

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

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Background Theory - filter (audio amplifier)

The transfer function

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

has the following properties:

- single pole at $p_1 = -\frac{1}{R_1C_1}$ and found by solving $R_1C_1s + 1 = 0$.
- single zero at $z_1 = -\frac{1}{(R_1 + R_2)C_1}$ and found by solving $(R_1 + R_2)C_1s + 1 = 0$.
- DC-gain of 1 and found by substitution $s = 0$ in $G(s)$. Related to the final value theorem for a step input signal $v_{in}(t)$:

$$\lim_{t \rightarrow \infty} V_{out}(t) = \lim_{s \rightarrow 0} s \cdot V_{out}(s) = \lim_{s \rightarrow 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s \rightarrow 0} G(s)$$

where $\frac{1}{s}$ is the Laplace transform of the step input $v_{in}(t)$.

- High frequency gain of $\frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$ and found by computing $s \rightarrow \infty$.

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Background Theory - filter (audio amplifier)

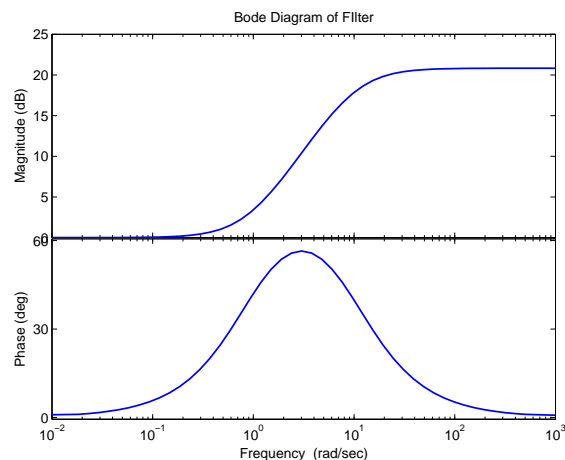
The transfer function

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

is a first order system where zero $z_1 = -\frac{1}{(R_1 + R_2)C_1} < p_1 = -\frac{1}{R_1C_1}$. This indicates $G(s)$ is a lead filter.

Easy to study in Matlab:

```
>> R2=100e3;R1=10e3;C1=10e-6;  
>> G=tf([(R1+R2)*C1 1],[R1*C1 1]);  
>> bode(G)
```



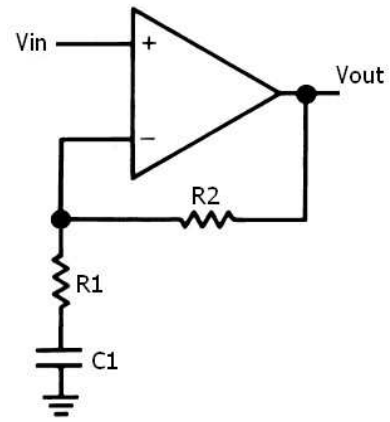
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Background Theory - filter (audio amplifier)

Amplification lead filter circuit with

$$V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1 + \frac{1}{jC_1\omega}} \right) V_{in}(j\omega)$$

will be used to strongly amplify a small high frequent signal but maintain (follow) the DC-offset.



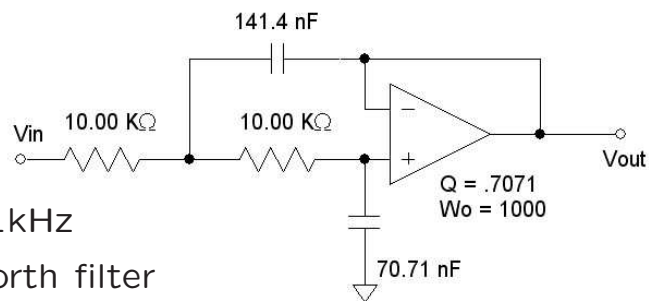
From the previous analysis we see:

- Gain at DC ($\omega = 0$) is simply 1.
- Gain at higher frequencies approaches $1 + \frac{R_2}{R_1}$

Background Theory - filter (2nd order Butterworth)

Another fine filter:

- 2nd order low pass **Butterworth filter**
- Pass-band frequency of 1kHz
- 2nd order 1kHz Butterworth filter is a standard 2nd order system



$$V_{out}(s) = G(s)V_{in}(s)$$

where

$$G(s) = \frac{\omega_n^2}{s^2 + 2\beta\omega_n s + \omega_n^2}$$

with $\omega_n = 2\pi \cdot 1000$, $\beta = \sqrt{1/2} \approx 0.707$.

Background Theory - filter (2nd order Butterworth)

$$V_{out}(s) = G(s)V_{in}(s)$$

where

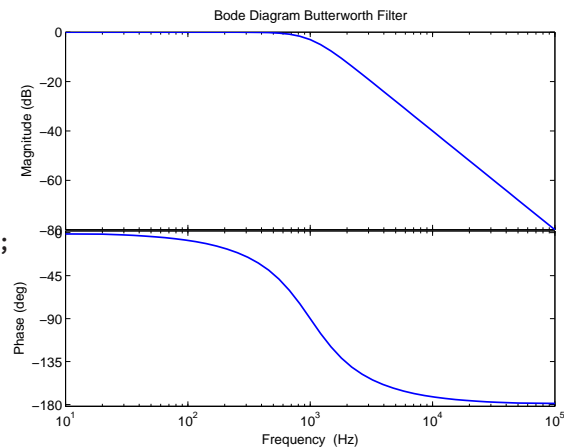
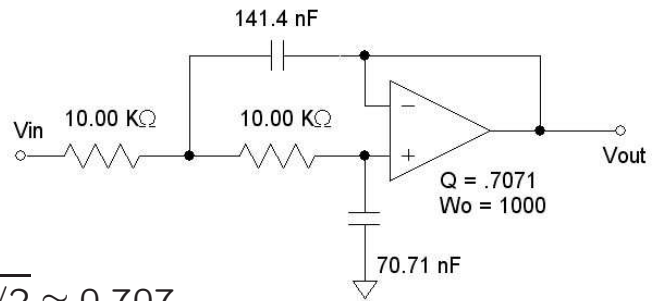
$$G(s) = \frac{\omega_n^2}{s^2 + 2\beta\omega_n s + \omega_n^2}$$

with $\omega_n = 2\pi \cdot 1000$, $\beta = \sqrt{1/2} \approx 0.707$

means:

- well damped filter
- -40dB/dec above 1kHz

```
>> [num,den]=butter(2,2*pi*1000,'s');
>> G=tf(num,den);
>> bode(G)
```



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Background Theory - simulation

For some of the circuits we can write down transfer functions (see for example Slide 32 and 35)

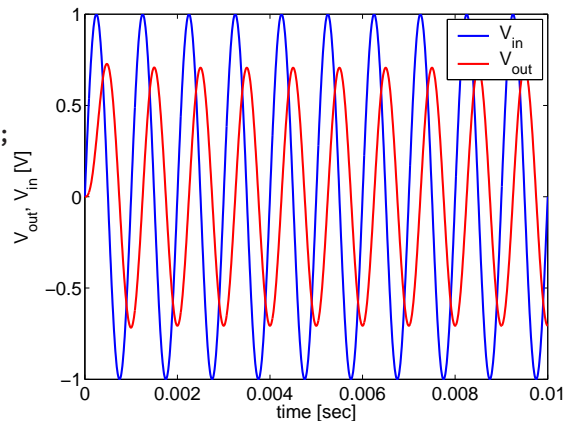
As a result you can also do a *linear* time domain simulation of your circuit via Matlab:

Example Matlab code for our Butterworth filter in Task 3-1:

```
>> [num,den]=butter(2,2*pi*1000,'s');
>> G=tf(num,den)
>> t=linspace(0,0.01,1000);
>> Vin=sin(2*pi*1000*t);
>> Vout=lsim(G,Vin,t);
>> plot(t,Vin,t,Vout,'r')
```

Drawbacks:

- only linear simulation
- you would have to derive the 'transfer function' for each signal in your circuit



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Background Theory - simulation

- SPICE (Simulation Program with Integrated Circuit Emphasis) is an open source electronic circuit simulator.
- Originated from Electronics Research Laboratory of the University of California, Berkeley by Laurence Nagel

Key to circuit simulation is a **good GUI** that allows you:

- Draw, edit and modify circuits
- Uses (non-linear) models of components
- Easily load new models of components
- Has good interface with SPICE to perform DC-, AC- and transient-analysis of your circuit

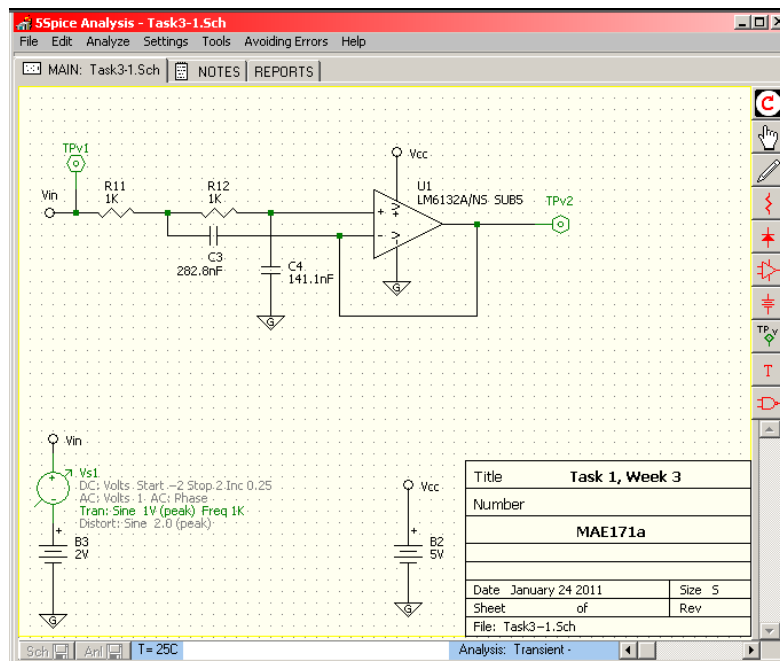
Good (limited version) for the PC (Windows) is [5Spice](#).

Optional, but try to use it for your circuit simulations!

We provide links to circuits already drawn for [5Spice](#).

Background Theory - simulation

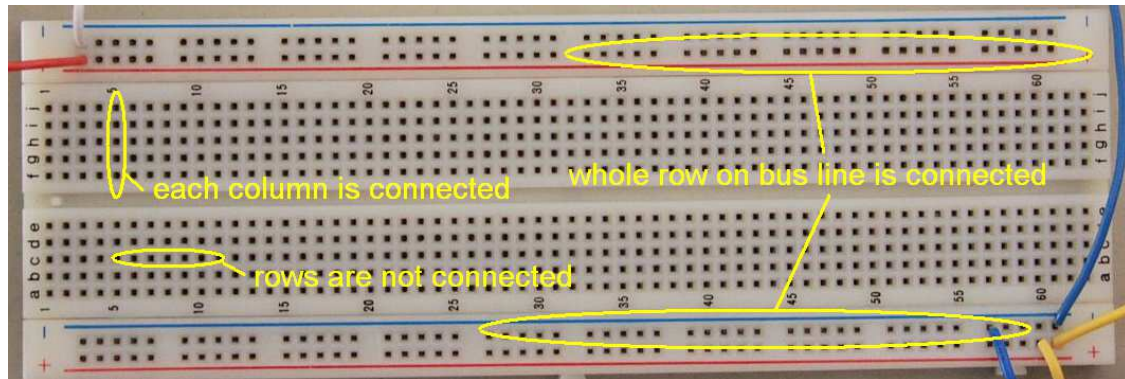
Our Butterworth filter demo in [5Spice](#):



Laboratory Work - breadboard

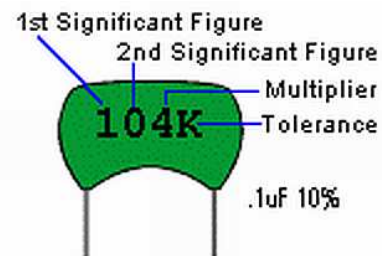
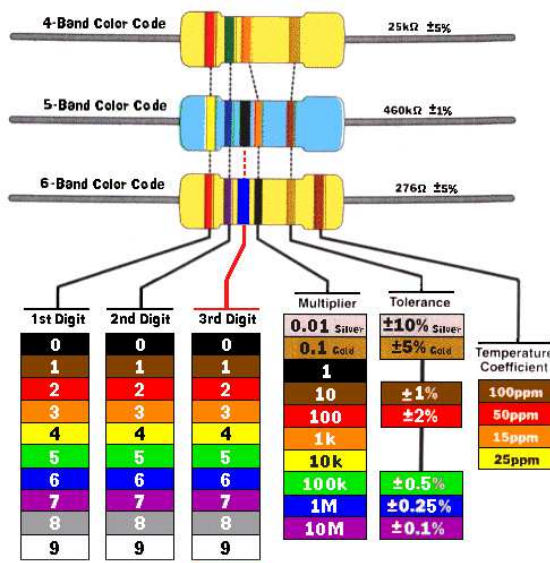
- Build/debug/test circuits on a *breadboard* or *protoboard*: a construction base for (prototype) electronic circuit.
- Allows you to connect components via *tie points* and avoid soldering (*solderless breadboard* or *plugboard*)
- Although useful for prototyping, mechanical connection in tie points sometimes fails. . . Hard to debug when it fails.

Important for our experiment: important to know how tie points are connected on breadboard:



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Laboratory Work - Resistor and Capacitor coding

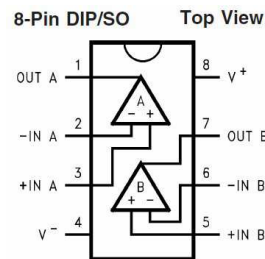
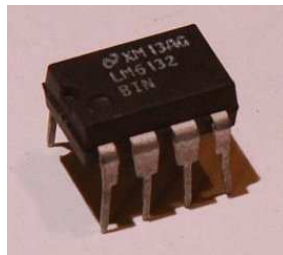
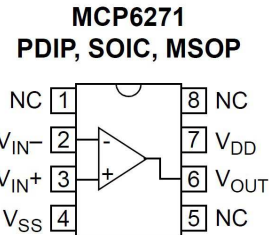
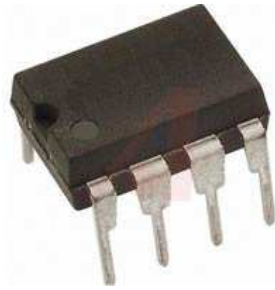


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Laboratory Work - rail-to-rail OpAmp

Rail-to-rail OpAmp used during laboratory work can be either:

- MCP6271 (single OpAmp in 8 pin package)
- LM6132 (dual OpAmp in 8 pin package)



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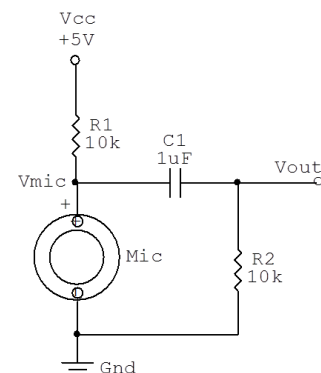
Laboratory Work - week 1

Two independent tasks that are combined in a final task 1-c:

- 1-a: create/test signal conditioning for electret microphone
- 1-b: create/test non-inverting amplifier

Task 1-a:

- Audio application: generate input signal via MIKE-74 electret microphone
- build a DC-bias circuit for the microphone to measure (sound) pressure variations
- Measure the DC-bias (offset) voltages
- Display and analyse the time plots generated by the microphone
- Compare experiments and [task 1-a simulation with 5SPICE](#).

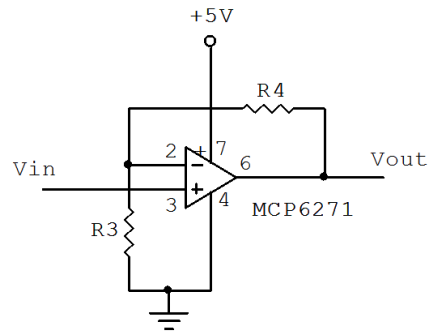


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Laboratory Work - week 1

Task 1-b (independent of task 1-a):

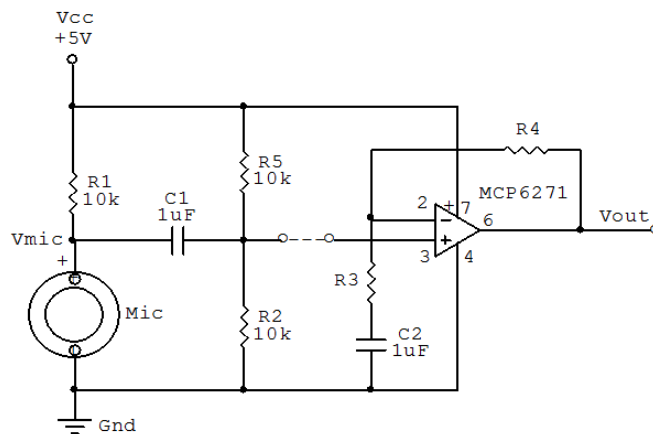
- Build non-inverting amplifier using op-amp
- Measure bias voltages
- Experiments:
determine gain for different resistor values and avoid clipping on output signal V_{out} .
- Measure the frequency response of your amplifier.
- Compare experiments and [task 1-b simulation with 5SPICE](#).



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Laboratory Work - week 1

Task 1-c: connect non-inverting amplifier to microphone signal conditioning circuit



- Additional resistor $R5$ to off-set of microphone signal.
- Additional capacitor $C2$ in series with $R3$ to create desired **audio amplifier** (see earlier slides in these lecture notes).
- Change $R3$ to 1K and $R4$ to 100K. Result: DC-gain of 1 and (high frequency) gain of 101.

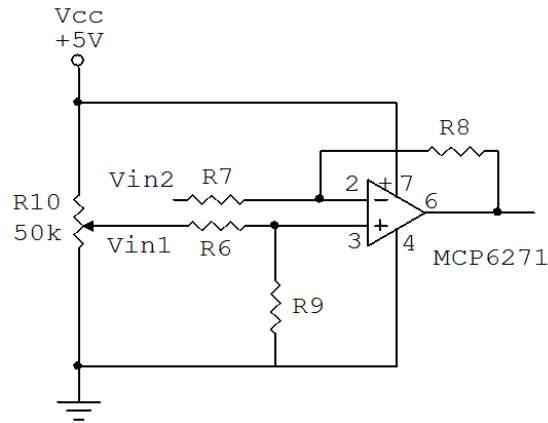
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Laboratory Work - week 2

Designing a mixer (difference amplifier) and combining it with the Week 1 circuit (microphone + audio amplifier).

Task 2-a: difference (differential) amplifier:

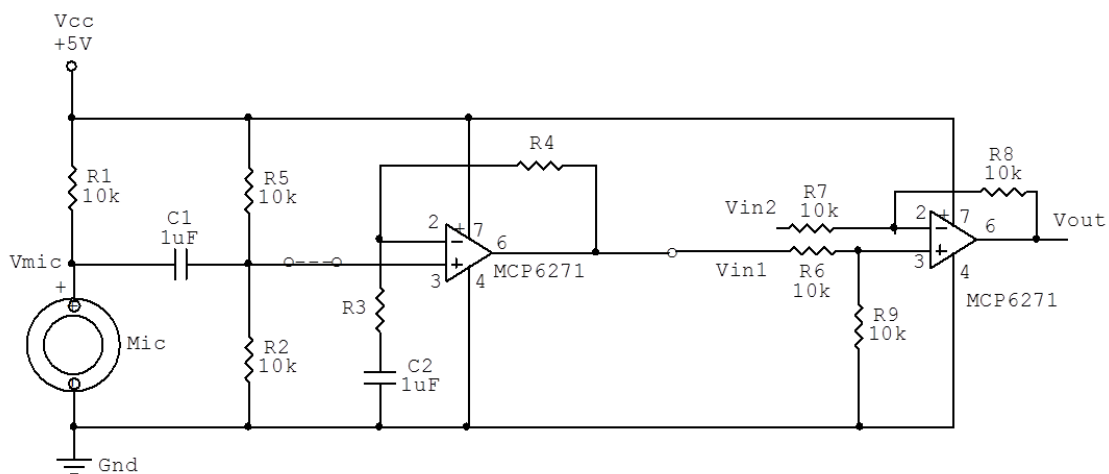
- Build/test the mixer (difference amplifier)
- Verification of operation (difference of signals)
- Bias voltage adjustment via pot meter R10.
- Gain adjustments and experimental verification of gain and linearity of circuit
- Compare experiments and [task 2-a simulation with 5SPICE](#).



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Laboratory Work - week 2

Task 2-b: combine microphone + audio amplifier from Week 1 with difference amplifier to allow mixing of signals.



- Connect Vin1 to Vout of Week 1 circuit - elimination of pot meter R10. Why?
- Verify operation of circuit - do signals of microphone nicely mix with signals applied to Vin2 without 'clipping'.

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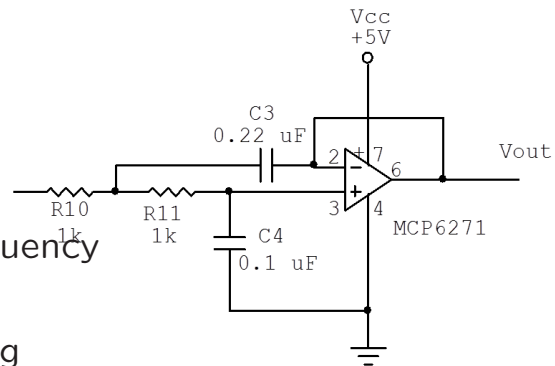
Laboratory Work - week 3

3-a: Create 2nd order Butterworth filter

3-b: Combine/test all parts of your circuitry

Task 3-a:

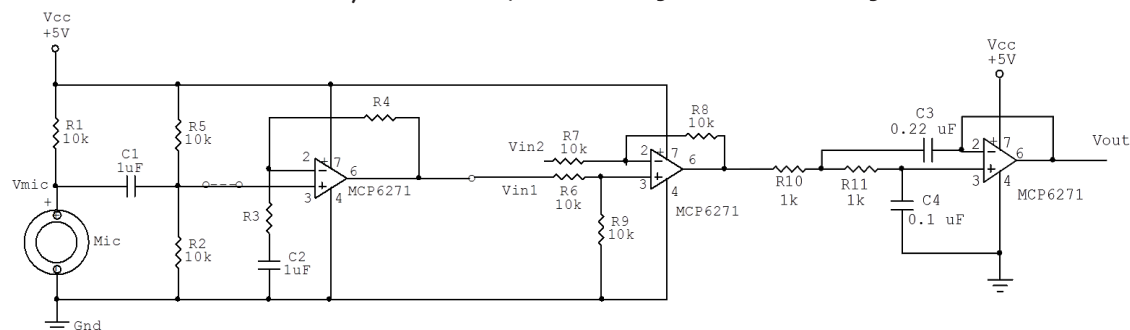
- Create active second order low pass filter with cut-off frequency around 1kHz (for speech).
- Demonstrate filter by measuring amplitude of output signal for sine input with different frequencies
- Compare frequency measurements with Bode plot in Matlab (see Slide 35 of this lecture)
- Compare experiments and [task 3-a simulation with 5SPICE](#).



MAE171a Linear Circuit Experiment, Winter 2014 – R.A. de Callafon – Slide 47

Laboratory Work - week 3

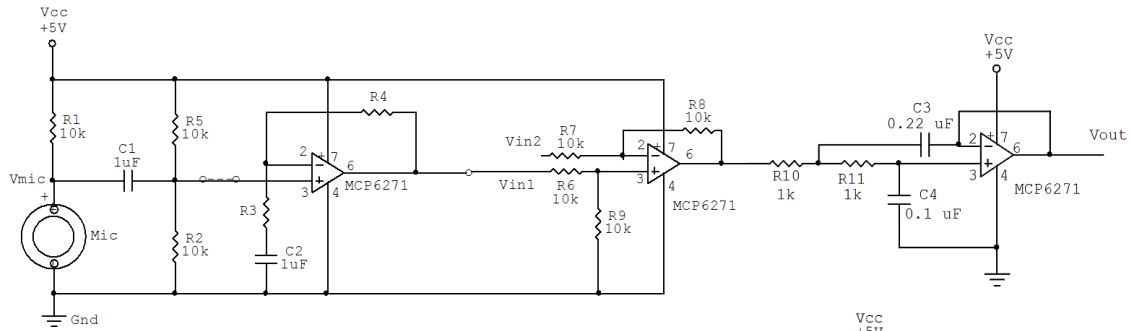
Task 3-b: Combine/test all parts of your circuitry



- All circuits combined.
- Should be able to filter and amplify microphone signal.
- Should be able to mix in and filter an addition signal at Vin2.
- Should be able to hear mixing (microphone and signal at Vin2) via Vout via set of additional capacitor and headphone.
- Measure signal at different locations in your circuit for your report to show 'clipping'.

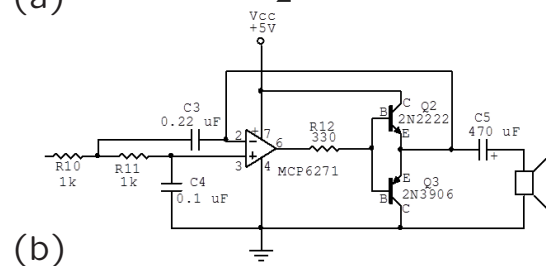
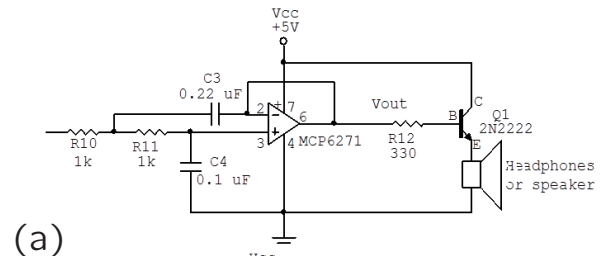
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Laboratory Work - week 3



Optional:

- Add a power boost to your circuit
- Power boost can be
 - (a) single NPN transistor
 - (b) double 'push-pull' or NPN/PNP transistor pair
- Attach speaker to hear result



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Summary

- (relatively simple) signal conditioning algorithms: *amplification*, *adding/difference* and basic (most 2nd order) *filtering*
- audio application on a single voltage power supply for amplification and filtering while maintaining DC (off-set)
- **challenge**: single source power supply of 5 Volt. **Avoid clipping/distortion of amplified, mixed and filtered signal.**
- insight in op-amp based linear circuits by build/debug
- compare theory (ideal op-amp) with practice (build and test)
- experimentally verify signals, gain and filtering of circuitry
- optional: compare experiments with simulation generated by [5SPICE simulation software](#)
- for error/statistical analysis: measure signals multiple times and estimate circuit parameters (gain, cut-off frequency) from experiments

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What should be in your report (1-2)

- Abstract
 - Standalone - make sure it contains clear statements w.r.t motivation, purpose of experiment, main findings (numerical) and conclusions.
- Introduction
 - Motivation (why are you doing this experiment)
 - Short description of the main engineering discipline (circuit design, amplification, filtering)
 - Answer the question: what is the aim of this experiment/report?
- Theory
 - Summary of relevant Op-Amp circuits to analyze your data
 - Summary of filtering, frequency response analysis
 - Summary on how you did simulation of your circuitry

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What should be in your report (2-2)

- Experimental Procedure
 - Short description of experiment
 - How are experiments done (detailed enough so someone else could repeat them)
- Results
 - Measured input/output signals to demonstrate microphone/amplification/filtering)
 - Frequency Response estimation
 - Effect of rail-to-rail conditions (saturation)
 - Parameter estimation (gain, cut-off frequency, etc.)
- Discussion
 - Why are simulation results different from experiments?
 - Do the Op-Amps behave as expected?
- Conclusions
- Error Analysis
 - Mean, standard deviation and 99% confidence intervals of estimated parameters from data
 - How do errors propagate?

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GOOD LUCK

