Heat Transfer From A Heated Vertical Plate

Mechanical and Environmental Engineering Laboratory
Department of Mechanical and Aerospace Engineering
University of California at San Diego
La Jolla, California 92093-0411

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Instructor: Kal Seshadri

HEAT TRANSFER FROM A HEATED PLATE IN A DUCT

In the following we will consider the heat transfer from a vertical heated plate. The aim is to determine the heat transfer coefficient h and from that the dimensionless form which is the Nusselt number Nu.

The surface of the plate is kept at a fixed temperature and air flows past the plate.

Objective

To measure the heat flux as a function of the flow and the plate temperature

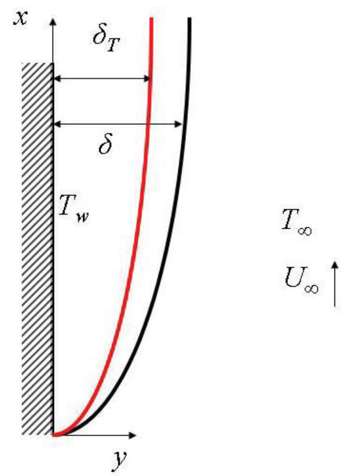

Figure 1: Defintion sketch of a heated plate.

Objectives

- Week 1
  1. To calibrate the heat flux meter
  2. To measure the velocity profile across the duct
- Week 2
  1. Measure the heat flux in forced convection, and free convection
  2. Determine the heat transfer coefficient (h), Nusselt number (Nu).
  3. Plot Nusselt numbers (Nu) against Reynolds number (Re) for forced convection, and against Raleigh Number (Ra) for free convection.
- Week 3
  1. Complete any unfinished experiments from the flat plate heat transfer experiment from previous weeks
  2. Determine the heat transfer coefficients for the fins
  3. Plot Nusselt numbers (Nu) against Reynolds Number (Re), or Rayleigh Number (Ra)

Outline

- Review of basic heat transfer
- Laminar forced convection
- Turbulent forced convection
- Free convection

Mechanisms of Heat Transfer

- Conduction: Requires a medium. Medium is stationary
- Convection: Requires a medium. There is flow.
  - Forced convection: Flow is driven by an external device
  - Free (natural) convection: Flow induced by buoyancy forces
- Radiation: No medium.

Conduction

- Heat transported through a stationary medium (e.g. a crystal lattice) by vibrational energy of molecules that increase with temperature.

- Fouriers law: Conductive heat flux vector $\mathbf{q}$ (units W/(m^2) is proportional to the temperature gradient. The constant of proportionality is the thermal conductivity k (units W/(m·K). Thus

  $$\mathbf{q} = -k\nabla T$$

  where ∇ is the gradient operator and T the temperature

Convection

- Heat transported through a moving medium (e,g. a fluid) by motion of the medium.

- In general there is a solid surface, with the fluid flowing over the solid surface. The heat transport from the solid surface to the moving medium is

  $$q = -k\left(\frac{\partial T}{\partial n}\right)_{n=0}$$

  where q the local heat flux, k the thermal conductivity of the moving fluid, n is the coordinate normal to the surface of the plate.

- the value of $(\partial T/\partial n)_{n=0}$ depends on the characteristics of the flow.

Convection

- Convective flux of heat transfer is generally expressed in terms a convective heat transfer coefficient h [units W/(m^2·K)] or a Nusselt Number Nu_{L_c} [dimensionless].

- The Newton's law of cooling is employed to define h and Nu. Thus

$$q \equiv h\left(T_w - T_\infty\right) \equiv Nu_{L_c}\frac{k}{L_c}\left(T_w - T_\infty\right)$$

where L_c is some characteristic length, T_w local value of temperature at the interface between the solid surface and the fluid, and T_∞ is the "free stream' temperature of the fluid. In view of

$$q = -k\left(\frac{\partial T}{\partial n}\right)_{n=0}$$

it follows that

$$h = -\frac{k}{T_w - T_\infty}\left(\frac{\partial T}{\partial n}\right)_{n=0}$$

and

$$Nu_{L_c} = -\frac{L_c}{T_w - T_\infty}\left(\frac{\partial T}{\partial n}\right)_{n=0} = \frac{hL_c}{k}$$

Describing Equations

- Equations
  - Conservation of Mass: Scalar equation
  - Equation of motion (Newton's Second Law)
  - Conservation of Energy (First law of Thermodynamics)
  - Equation of State.

- Independent and Dependent Variables
  - Independent variables: time (t), spatial coordinates, (x, y, z)
  - Unknowns (dependent variables): density (ρ), pressure distribution (p), three components of flow velocity (u, v, w), temperature (T). Total number of unknowns six.
  - Number of equations six.

- Solution Strategy
  - Integrate the equations and apply the initial conditions and boundary conditions to obtain the velocity distribution and temperature distribution. Note $T(t, x, y, z)$
  - Calculate heat flux using $q = -k\left(\partial T/\partial n\right)_{n=0}$

Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \overline{w})$$

where $\overline{w} = \overline{i}u + \overline{j}v + \overline{k}w$. With $\overline{i}, \overline{j}$, and $\overline{k}$, being unit vectors along the x, y, and z coordinates.

- Consider co-ordinate x parallel to plate, co-ordinate y normal to the plate. For a vertical plate the direction of the gravitational force is parallel to the plate in the $-\overline{i}$ direction.

- For constant density (but unsteady), and two-dimensional

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Equation of Motion

$$\rho \frac{\partial \overline{w}}{\partial t} + \rho (\overline{w} \cdot \nabla) \cdot \overline{w} = -\nabla p + \nabla \cdot \overline{\overline{\tau}} + \rho \overline{F}$$

where $\overline{\overline{\tau}}$ is the stress tensor and $\overline{F}$ is the body force per unit mass.

- Consider steady, two dimensional flow with constant viscosity. Employ the so-called "Boussinesq approximation" where changes in density are considered only for estimating the buoyancy forces, otherwise density is constant. Thus the equation of motion in the x direction parallel to the plate is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g \frac{T - T_\infty}{T_\infty}$$

where ν is the kinematic viscosity (units m^2/s), and g acceleration due to gravity.

Conservation of Energy

$$\rho c_{\mathrm{p}} \frac{\partial T}{\partial t} + \rho c_{\mathrm{p}} \left(\overline{w} \cdot \nabla T \right) = -\nabla \cdot \mathbf{q} + \left(\frac{T}{V} \frac{\partial V}{\partial T} \right)_p \frac{DP}{Dt} + \mu \phi_v$$

- Consider low speed, steady, two dimensional flow with constant density, viscosity and thermal conductivity. The energy equation is

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

where $\alpha = k / \left(\rho c_{\mathrm{p}} \right)$ is the thermal diffusivity (units m^2/s).

Nondimensionalization

Introduce the nondimensional quantities

- Independent variables

$$x^\star = x / L_c; \;\; y^\star = y / L_c;$$

- Velocities

$$u^\star = u / U_\infty; \;\; v^\star = v / U_\infty;$$

- Temperature

$$T^\star = \left(T - T_\infty \right) / \left(T_w - T_\infty \right)$$

Nondimensional Numbers

Introduce the nondimensional quantities

- Reynolds Number

$$Re = \frac{U_\infty L_c}{\nu}$$

- Prandtl number

$$Pr = \frac{\nu}{\alpha}$$

- Grashof number

$$Gr = \frac{g L_c^3 \left(T_w - T_\infty\right)}{\nu^2 T_\infty}$$

Describing Equations Employing Boundary Layer Approximations

- Mass Conservation

$$\frac{\partial u^\star}{\partial x^\star} + \frac{\partial v^\star}{\partial y^\star} = 0$$

- Equation of Motion

$$u^\star \frac{\partial u^\star}{\partial x^\star} + v^\star \frac{\partial u^\star}{\partial y^\star} = \frac{1}{Re} \frac{\partial^2 u^\star}{\partial y^{\star 2}} + \frac{Gr}{(Re)^2} T^\star$$

- Conservation of Energy

$$u^\star \frac{\partial T^\star}{\partial x^\star} + v^\star \frac{\partial T^\star}{\partial y^\star} = \frac{1}{RePr} \frac{\partial^2 T^\star}{\partial y^{\star 2}}$$

Note:

- Forced Convection when $Gr \ll (Re)^2$
- Free Convection when Gr is of the same order of $(Re)^2$

Results of Boundary Layer Analysis

- Forced Convection
  - At any location (local)

    $$Nu_x = 0.332\,(Re_x)^{1/2}\,Pr^{1/3}$$

    where the characteristic length employed in the definition of Nusselt Number and Reynold Number is $L_c = x$
  - Average Nusselt Number

    $$Nu_L = 0.664\,(Re_L)^{1/2}\,Pr^{1/3}$$

    where the characteristic length employed in the definition of Nusselt Number and Reynold Number is the length the plate $L_c = L$

- For Free convection, the empirically obtained average Nusselts Number is

  $$Nu_L = 0.68 + \frac{0.670(Ra_L)^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

  where the Raleigh Number $Ra = Gr \times Pr$.

The experiments

The objectives of the experiments are

1. To calibrate the apparatus
2. To measure the heat transfer from the plate
3. To compare the results with theory

The experiment consists of heated plate in an air flow. Heat is supplied to the plate to maintain it at a given temperature. By measuring the power required to maintain the plate temperature the heat flux is determined. The aim is to measure the *heat transfer coefficient* for a range of temperatures and air flow rates, covering *free convection, laminar and turbulent forced convection*.

A labview VI controls the heater so that the temperature rapidly reached the set value. Once this is reached the system is set to manual and the heater is maintained at a fixed power. The heat transfer coefficients and the Nusselt number are determined from the measurements of the power and the temperatures if the air and the plate.

Figure 3: An overview showing the duct and the control electronics.

Figure 4: A close-up showing the heated plate and the velocity probe.

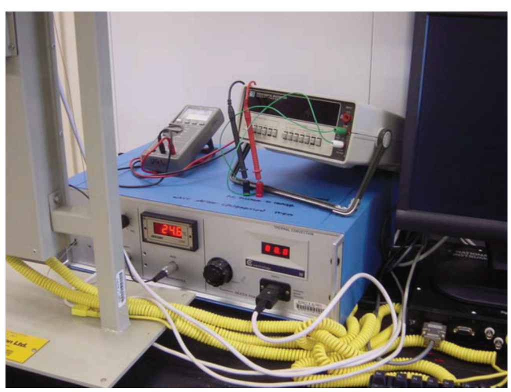

Figure 5: A close-up of the control unit.

1. Week 1 *Calibration*

Objective

To calibrate the power meter and measure the velocity profile

Method

The power to the plate is set by a control knob with a panel display. Measure the voltage and current drawn by the power supply and calibrate the panel display from 0 – 20 W.

Measure the velocity profile across the duct at different flow speeds.

Weeks 2 and 3 *Heat flux measurements*

Objective

To measure the heat flux to the plate for different plate temperatures and flow speeds in the duct

To compare the results with theory

Method

Set the flow rate and plate temperature to desired values. Measure the power to the plate.

Plot the results of the Nusselt number against the flow parameters and check the theoretical results.

REFERENCES

LIENHARD, J.H. 1987 *A heat transfer textbook.* Prentice-Hall.