Use of a Mariotte bottle for the experimental study of the transition from laminar to turbulent flow

J. A. Maroto[a)] and J. de Dios[b)]
Department of Physics, Polytechnic School, University of Jaén, 23700 Linares, Jaén, Spain

F. J. de las Nieves
Department of Applied Physics, University of Almería, Almería, Spain

(Received 22 January 2001; accepted 20 February 2002)

A Mariotte bottle is a device that provides a constant effusion velocity for liquids. We discuss a Mariotte bottle that has been designed to study the flow regime and the transition from laminar to turbulent flow. Several straight, smooth, circular glass tubes with different lengths and sections were inserted into the lower part of the bottle so that the rate of flow could be measured in very different experimental conditions by using a precision balance. Reynolds numbers in the interval $1014 \leqslant R \leqslant 6098$ were obtained, showing that the flow regime is laminar for $R \leqslant 3000$. There is a transition in the flow regime for R in the interval 3000–4000 and a turbulent flow regime for higher R. Because the device is very simple and the results obtained are very clear and exemplary, we recommend using this device as a laboratory experiment for physics or engineering students who require a knowledge of fluid mechanics.
[DOI: 10.1119/1.1469038]

I. INTRODUCTION

Although the theory of laminar flow is well developed and many solutions are known,[1,2] there is no theoretical analysis or computer solution that can simulate the small scale random fluctuations of turbulent flow. Thus, the theory for turbulent flow is semiempirical and is based on dimensional analysis and physical reasoning that permits us only to evaluate the average properties and the variance of the fluctuations. Nevertheless, the theory of turbulent flow is surprisingly effective.

It is of much practical interest for engineers and physicists to have prior knowledge of the flow regime (laminar or turbulent) that will govern the movement of a fluid. In 1883, Osborne Reynolds demonstrated that the nature of the flow regime depends on the value of a certain dimensionless parameter that represents the ratio between the inertial force and the viscosity.[3] This parameter R, known as the Reynolds number, has been widely used. It has been experimentally demonstrated that for the flow of fluids through circular tubes, there is a critical value of R of approximately 2300 that marks the transition from one flow regime to another.

To analyze the transition regime from laminar to turbulent flow, we have used a Mariotte bottle that we designed and constructed. A Mariotte bottle is a device that provides a constant effusion velocity for liquids. Historically, the Mariotte bottle was frequently used in the 19th century in oil lamps for domestic illumination.[4] It consists of a closed bottle whose cover is crossed by a vertical tube. To achieve a constant effusion velocity, the lower end of the bottle must be constantly submerged in the liquid that fills the bottle.

In this work we have inserted several straight, smooth, circular glass tubes with different lengths and diameters in a hole in the lower part of the bottle's wall, and have investigated the flow regime of water by means of simple flow rate measurements. The velocity of the liquid that pours out of the tube versus the distance between the lower end of the vertical tube and the level of the horizontal tube was determined for each tube. The experimental Reynolds numbers were then determined and compared with the theoretical values calculated by means of the formulas for laminar and turbulent flows.

The use of glass tubes of different but similar lengths and diameters allowed us to check that the transition from laminar to turbulent flow is independent of these parameters, and depends, for a given material (glass) and liquid (water), only on the Reynolds number.

II. THEORY

Bernoulli's law for the viscous flow of a liquid between two points i and j of a tube takes the following form:[5]

$$\left(\frac{p_i}{\rho}+\frac{v_i^2}{2}+gy_i\right)-\left(\frac{p_j}{\rho}+\frac{v_j^2}{2}+gy_j\right)=H, \tag{1}$$

where p is the pressure, v is the velocity of the flowing liquid, y is the height with respect to an arbitrary reference level, g is the acceleration of gravity, and H is the loss of head along the current tube, that is, the energy loss due to the increase of the specific internal energy of the liquid and the heat dissipated.

Let us apply Eq. (1) to the configuration schematically shown in Fig. 1, where, besides the characteristic dimensions of the container, the length L and the diameter D of the horizontal tube are shown, as well as the distance h between the lower end of the vertical tube and the horizontal tube and the three characteristic points 0, 1, and 2. Bernoulli's law for points 1 and 2 of Fig. 1 has the following form [with $v_1 = v_2 = v$, $y_1 = y_2 = 0$ and $p_2 = p_a$ (atmospheric pressure)]:

$$\frac{p_1 - p_a}{\rho} = H_L + H_l, \tag{2}$$

where H_L is the loss of head for the liquid flow from point 1 to 2, and H_l is the loss of head for the liquid intake and the liquid outlet in the horizontal tube (points 1 and 2, respectively).

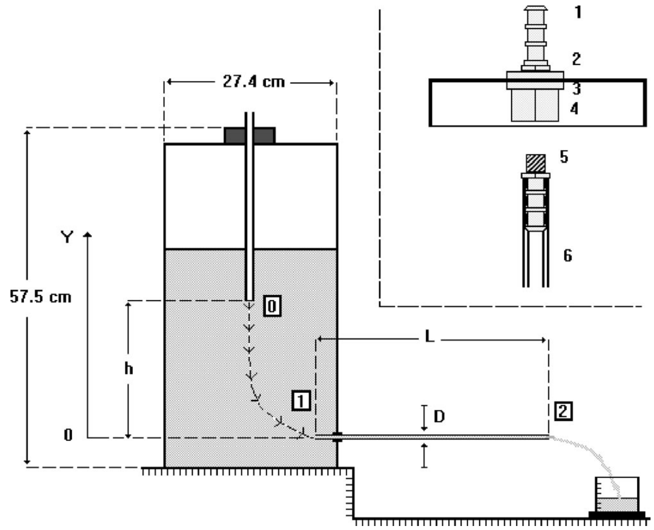


Fig. 1. Experimental assembly, characteristic dimensions of the container, and details of the lid, where each of the polypropylene tubes can be screwed and unscrewed.

Although H_L depends on the flow regime (laminar or turbulent), a variety of experimental data suggest that H_l is independent of the flow regime. Most authors use the following law for both points 1 (intake) and 2 (outlet):

$$H_l = K\frac{v^2}{2}, \tag{3}$$

where K is a constant known as the friction coefficient. We have used the commonly accepted values of $K=0.78$ in the intake and $K=1$ in the outlet.[5–7]

On the other hand, Bernoulli's law for points 0 and 1 of a streamline, taking into account that $y_0 - y_1 = h$, $p_0 = p_a$, $H = 0$, and that $v \sim 0$, can be expressed as:

$$\frac{p_a - p_1}{\rho} - \frac{v^2}{2} + gh = 0. \tag{4}$$

A. Laminar flow

We next evaluate the term H_L of Eq. (2), which depends on the flow regime. For laminar flow, H_L is easily obtained from the Hagen–Poisseuille law:[5]

$$H_L = \left(\frac{64}{R}\right)\frac{L}{D}\frac{v^2}{2}, \tag{5}$$

where $R = \rho D v/\eta$ is the Reynolds number, and η is the liquid viscosity.

By substituting Eqs. (5) and (3) in Eq. (2) and combining Eqs. (2) and (4) to eliminate the common term $(p_1 - p_a)/\rho$, we obtain:

$$2.78D^2\rho v^2 + 64L\eta v - 2ghD^2\rho = 0. \tag{6}$$

B. Turbulent flow

In this case the loss of head H_L can be evaluated by means of the empirical formula of Blasius,[6] which is valid for smooth tubes and Reynolds numbers up to 10^5:

$$H_L = \frac{0.158Lv^2}{DR^{1/4}}. \tag{7}$$

By carrying out the same steps as in the previous case, the following relation can be found:

$$(2.78\rho^{1/4}D^{5/4})v^{9/4} + (0.316\eta^{1/4}L)v^2 - (2gh\rho^{1/4}D^{5/4})v^{1/4} = 0. \tag{8}$$

Both Eqs. (6) and (8) can be expressed in the form:

$$F(\rho,\eta,L,D,v,h) = 0. \tag{9}$$

Given that ρ and η depend only on the liquid (water) and the laboratory temperature (20 °C), we take $\rho = 10^3\ \mathrm{kg\,m^{-3}}$ and $\eta = 1.002\times 10^{-3}\ \mathrm{kg\,m^{-1}\,s^{-1}}$. Once a particular tube is inserted in the hole of the Mariotte bottle, the values of the length L and the diameter D are fixed, and Eq. (9) can be written as:

$$F(v,h) = 0. \tag{10}$$

Equation (10) shows that for each value of h, there is a value of the effusion velocity v that satisfies $F=0$.

For laminar flow, Eq. (10) takes the following form:

$$av^2 + bv + ch = 0, \tag{11}$$

where a, b, and c are constants. Therefore, the evaluation is reduced to the solution of a quadratic equation.

For turbulent flow, Eq. (10) has the form:

$$a'v^{9/4} + b'v^2 - c'hv^{1/4} = 0, \tag{12}$$

where a', b', and c' are constants. In this case the evaluation is more complex and requires, in general, the use of numerical methods to determine the value of v that satisfies Eq. (12) for a particular value of h.

III. DESIGN OF THE MARIOTTE BOTTLE AND SELECTION OF TUBES

To design and construct a useful Mariotte bottle, a container of large dimensions (length = 54.3 cm, diameter = 26.2 cm) was chosen and a hole was opened in the lower part of the wall (see Fig. 1). Several straight, smooth, and circular tubes of glass were inserted in the hole in order to empty it.

The container lid was perforated and two tightening joints (named 2 and 3 in Fig. 1) were pasted on it, carefully centering them in the hole. A wide nut was inserted in the lower joint (named 4) and to attach it to the lid, a spigot (named 1) was screwed on it. The spigot is hollow so that it permits air intake and outlet. Finally, a long polypropylene tube was cut in sections of lengths of 5–40 cm in order to insert the tubes in different spigots (see 5 and 6 in Fig. 1). This assembly permitted us to select the desired distance h by screwing the suitable polypropylene section onto the nut. Moreover, this assembly allowed the hermetic sealing of the container and the correct functioning of the Mariotte bottle.

Three glass tubes were selected with different and similar lengths and diameters (see Table I) to investigate if the L and

Table I. Lengths and diameters of the three glass tubes used to empty the container.

Tube	Length (cm)	Diameter (mm)
1	29.3±0.1	2.42±0.05
2	56.7±0.1	3.96±0.03
3	50.5±0.1	5.36±0.03

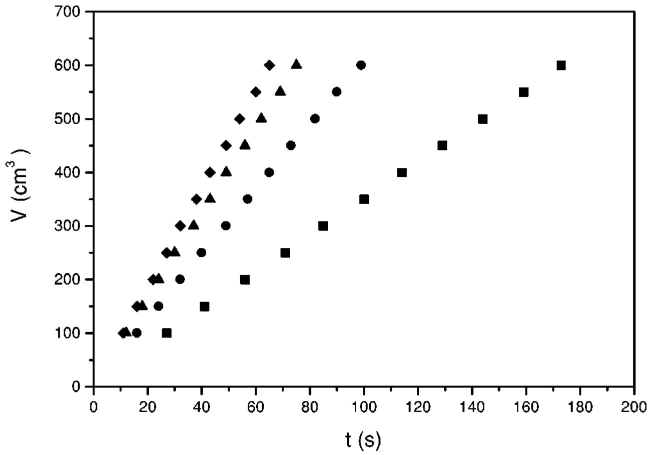


Fig. 2. Volume poured out vs the time for tube 2 and the following distances *h*: (◆) 20 cm, (▲) 15 cm, (●) 10 cm, (■) 5 cm.

D values of each tube influenced the transition from laminar to turbulent flow.

IV. RESULTS AND DISCUSSION

Some experiments were carried out to evaluate the effusion velocity v versus the distance h. The liquid of the container was poured out by means of one of the three glass tubes into a precipitate glass with a capacity of 600 ml, which was placed on an electronic balance (see Fig. 1). The balance had a precision of 0.1 g. The volume poured out versus the time was evaluated by using a chronometer and by knowing the water density at 20 °C to be 1 g cm^{-3}. Figure 2 shows the volume V poured out versus the time t for tube 2 and different distances h. The linear dependence of V vs t is a result of the constant effusion velocity and is a demonstration of the correct functioning of the Mariotte bottle. Similar curves were obtained for tubes 1 and 3.

Next, the rate of flow, I_v, was evaluated for each tube and distance h from the slope of the $V(t)$ curve. The linear dependence of the experimental $V(t)$ curves made the evaluation method easy and unambiguous. Finally, by using that $I_v = Av$, where A is the tube cross section, the experimental values for the effusion velocity v were obtained.

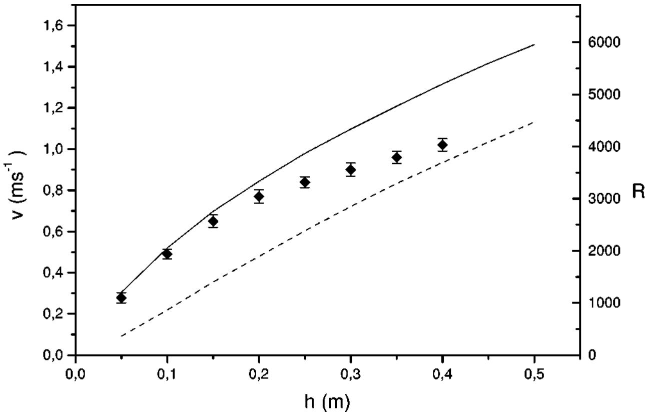


Fig. 4. Tube 2: experimental data for the effusion velocity v vs the distance h (◆) and theoretical data for the laminar (solid line) and turbulent (dotted line) flows.

Equations (11) and (12) were used to obtain the theoretical curves for the effusion velocity v versus the distance h for laminar and turbulent flows, respectively. As was discussed in Sec. III, the determination of v for laminar flow involves the solution of a second-order equation. For turbulent flow, we used a graphical method and plotted the left-hand side of Eq. (12) versus v. Given a particular value of h, the desired velocity v is the one that crosses the abscissa. The use of a commercial program, DERIVE, facilitated this solution.

Figures 3, 4, and 5 show the experimental and theoretical data for the effusion velocity v versus the distance h for tubes 1, 2, and 3, respectively. Figure 3 shows effusion velocities from 0.42 to 1.23 m/s and R values from 1014 to 2970. We see that there is excellent agreement between the theoretical data for laminar flow and the experimental data.

Figure 4 shows effusion velocities from 0.28 to 1.02 m/s and R values from 1098 to 4031. We found a reasonable fit between the theoretical data for laminar flow and the experimental data for R values up to 3000. For the larger values of R, neither the laminar nor turbulent theories are in agreement with the experimental data.

Finally, Fig. 5 shows effusion velocities from 0.41 to 1.14 m/s and R values from 2193 to 6098. We found that the theoretical data for laminar flow provided a reasonable fit with the two lower experimental values of v, that is, for R up to 3263. There also is an intermediate value of v (0.74 m/s)

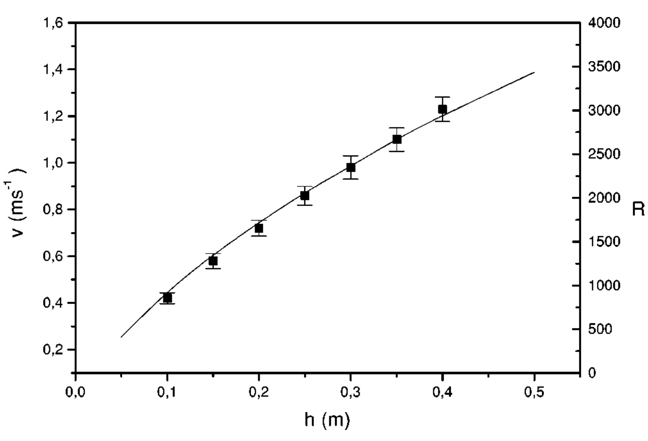


Fig. 3. Tube 1: experimental data for the effusion velocity v vs the distance h (■) and theoretical data for the laminar flow (solid line).

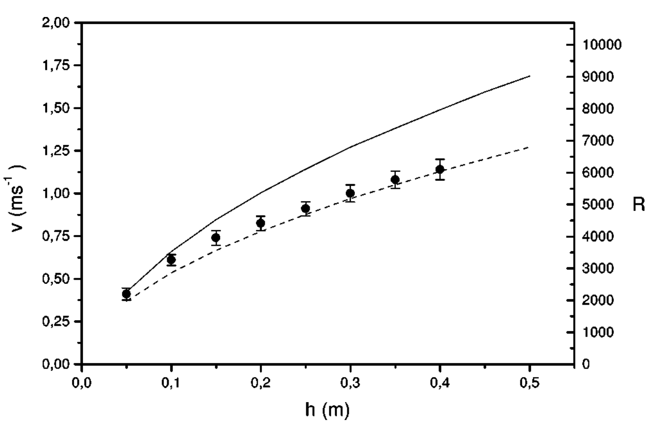


Fig. 5. Tube 3: experimental data for the effusion velocity v vs the distance h (●) theoretical data for the laminar and turbulent flows.

that is not in agreement with both theories. For $R>4407$, we found that the theoretical data for the turbulent flow are in agreement with the experimental data.

We conclude that the flow regime for the flow of a liquid (water) through straight, smooth, and circular glass tubes with different but similar lengths and diameters is laminar for R values up to approximately 3000 and turbulent for R above approximately 4400; there is an intermediate and broad interval of R values (from 3000 to approximately 4400) in which a transition of the flow regime takes place.

To evaluate the validity of our results, we note that the commonly accepted R value of 2300 marks the change of regime for the flow of fluids through circular tubes, but also that this critical value increases for smooth tubes.[2,7]

Finally, let us make it clear that the change of regime does not depend on the effusion velocity v, but on the Reynolds number R. For example, for tube 1 the flow is laminar for an effusion velocity of 1.2 m/s, and for tube 2 the flow is not laminar for an effusion velocity of 0.77 m/s (see Figs. 4 and 5).

V. SUMMARY

The use of a Mariotte bottle to empty a container by means of straight, smooth, circular glass tubes has permitted the design of a useful laboratory experiment that provides experimental evidence of the transition from laminar to turbulent flow.

Because the device is very simple and the results obtained are clear and unambiguous, we recommend this device for use as a laboratory experiment for students of fluid mechanics.

[a]Electronic mail: jamaroto@ujaen.es

[b]Present address: APCAR, Avda de Barcelona s/n, 23200 La Carolina, Jaén, Spain.

[1]H. Schlichting, *Boundary Layer Theory*, 6th ed. (McGraw–Hill, New York, 1968).

[2]F. M. White, *Viscous Fluid Flow* (McGraw–Hill, New York, 1974).

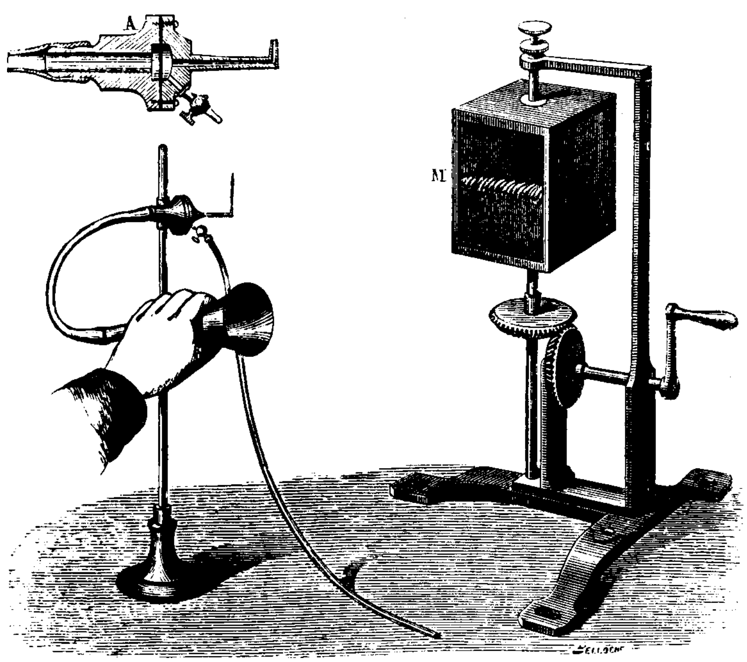
[3]O. Reynolds, "An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous and of the law of resistance in parallel channels," Philos. Trans. R. Soc. **184**, 935–982 (1883).

[4]M. Fernández de Fígares, *Manual de Física y Nociones de Química* (Libería de la Señora Viuda e Hijos de Zamora, Granada, 1873).

[5]R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics* (Wiley, New York, 1973).

[6]F. M. White, *Fluid Mechanics* (McGraw–Hill, New York, 1979).

[7]V. L. Streeter and E. B. Wylie, *Fluid Mechanics* (McGraw–Hill, New York, 1986).



Manometric Flame Apparatus. Gas flows into the manometric capsule at the top left and burns in a small flame. The supply of gas to the flame is modulated by acoustic signals impinging on the membrane in the middle of the capsule, causing the height of the flame to oscillate up and down. The oscillations of the flame are observed in the rotating mirror. This apparatus, invented by Rudolph Koenig of Paris in 1862, allowed the wave shape of a musical sound to be observed, and was the nineteenth century equivalent of a microphone-oscilloscope combination. This illustration is from Ganot's *Physics*, ca. 1875. (Notes by Thomas B. Greenslade, Jr., Kenyon College)