Position Control Experiment – MAE171a

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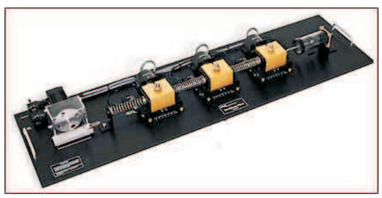

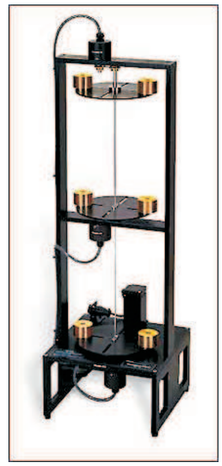

Contents

1 Aim and Procedure of Control Experiment

1.1 Aim of experiment

The rectilinear and torsional system are mechanical systems that are include mass or inertia, spring and damper elements. By configuring the torsional and rectilinear system in such a way that respectively 2 carts or 2 inertia disks are connected via spring elements (spring or flexible shaft), a two degree of freedom (2DOF) mechanical system is created where each mass or inertia has a positioning freedom.

> **The aim of this control experiment is to model and control the position/rotation of a mass/inertia of the 2DOF mechanical system.**

- The *modeling* is used to predict and model the flexibilities and vibrations in the 2DOF system. During the modeling phase of this experiment you may use the measurements of the position of both carts (encoder 1 and encoder 2).
- The *control* is used to change the position/rotation of a mass/inertia in the 2DOF mechanical *as fast as possible* without excessive residual vibrations of the mechanical system. During the control phase of this experiment you can only use the position of one cart for feedback control of the motor. You are free to choose which cart will be used in your feedback control and you must mention (in your report) which cart is used in your feedback control. When using cart 1 (encoder 1) for feedback, the type of control in this experiment is called co-located (as force location and position measurement are at the same location). When using the second cart (encoder 2) for feedback, the type of control in this experiment is called non co-located.

More background information on the dynamic modeling is given in Section 2. Laboratory experiments to estimate model parameters and validate the model are given in Section 3. Background information on the control design is given in Section 4. A summary on the contents of the laboratory report is given in Section 5.

1.2 Laboratory procedure

The experiment needs to be completed within 3 laboratory session of 3 hours each. During the experiment you are asked to formulate and validate a dynamic model of both the rectilinear and torisional system and develop a robust control algorithm that allows positioning of the two masses to a specific position with a limited overshoot and a fast settling time. Below is the list of tasks. More details on the theory can be found in the following sections.

- week 1
  - Introduction to `ECP` software used for experiments and controller implementation.
  - Read this handout and get familiar with the (modeling of the) 2DOF system.
  - Set up your system according to the `config.txt` file provided to you.

- Create list of open-loop (no control) step response experiments you will do to estimate the (unknown) parameters (m_1, m_2, k_1, k_2, d_1 and d_2) of the 2DOF system. This can be done by a sequence of open-loop step response experiments on a 1DOF system that has only a single mass m, damping d and stiffness k. The sequence of experiments must be such that it allow you to estimate all the parameters of the 2DOF system.
- Perform the 1DOF open-loop step response experiments several times (min. 5) and estimate mass m, damping d and stiffness k parameters from each 1DOF open-loop step response experiment.
- Validate your estimated parameters of the 1DOF model by comparing a measured and a simulated open-loop step response. This can be done by editing the `m1`, `d1` and `k1` parameters in the `parameters.m` file provided to you and running `maelab.m` script file for a 1DOF system.

- week 2
  - Set up your system according to the `config.txt` file provided to you.
  - Finish modeling of your 2DOF system by estimating and entering all the parameters m_1, m_2, k_1, k_2, d_1 and d_2 in the `parameters.m` file.
  - Before stating your control design, first validate the complete 2DOF model.
    * For rectilinear system: validate complete 2DOF model via comparison of a measured and a simulated open-loop step response by editing `parameters.m` and running `maelab` script file.
    * For torsional system: validate complete 2DOF model via inspection of resonance modes (Bode plot from `maelab` script file) and comparison of the location of resonance modes (sinusoidal excitation with ECP).
  - With a successfully validated model, use the `maelab` script file to determine the maximum value of k_p^{max} before closed-loop system becomes unstable. DO NOT implement this proportional controller on the real system, but design a P-controller with `maelab` script file that is stable and has a gain $k_p < k_p^{max}/2$.
  - Implementation and verification of P-controller with `ECP` software. Perform closed-loop step experiments with `ECP` and compare with closed-loop step simulations using the `maelab` script file.

- week 3
  - Set up your system according to the `config.txt` file provided to you.
  - With your successfully validated model (Week 2), design a PD or PID-controller (with `maelab` script file) satisfying the design requirements of no steady-state error and less than 25% overshoot.
  - Validate your controller by running closed-loop step simulations using the `maelab` script file and inspect steady-state error, overshoot and control signal saturation of ±5Volt when designing and validating the controller.

– Using ECP, measure and compare closed-loop step response experiments with your PD/PID controller and compare with closed-loop step simulations using the maelab script file.

– Compare differences between rectilinear and torsional system.

– Evaluation of the final control design via sensitivity analysis: measure change in overshoot and settling-time by performing at least 3 experiments in which mass of carts of rectlinear systems and location of weights (inertia) in torsional system has been changed.

2 Background Theory on Modeling

2.1 Modeling of 2DOF systems

2.1.1 Schematics and block diagram of experimental set up

A schematic diagram of the experimental set up of the torsional and the rectilinear system is depicted in Figure 1.

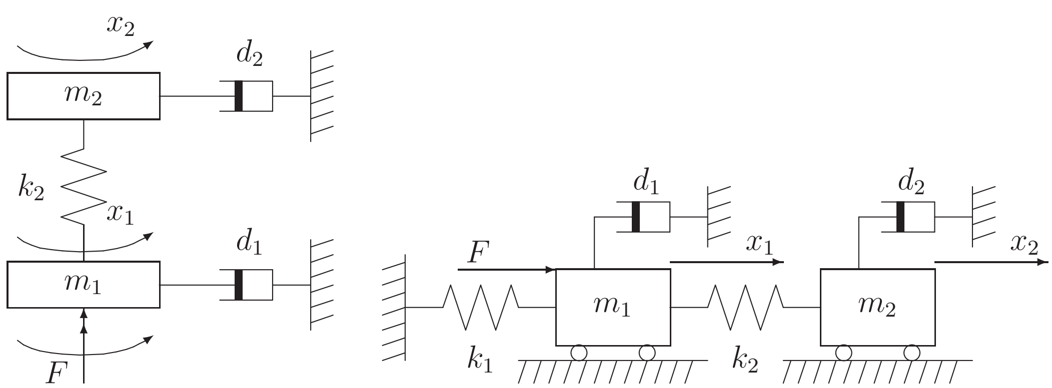

Figure 1: Schematic view of torsional system (left) and rectilinear system (right).

The 2DOF mechanical systems depicted in Figure 1 consist of 2 masses/inertia m_1, m_2 that each have a positioning freedom x_1, x_2. The masses are connected via spring elements k_1, k_2 (spring or flexible shaft). Additionally, to model the damping present in the 2DOF system, a viscous damping d_1, d_2 is assumed to act on each of the masses in the mechanical system.

The mechanical system can be moved and positioned by a *control force*, denoted by F in Figure 1. For the rectilinear system, the control force is a rectilinear force applied to the first mass m_1, while for the torsional system a momentum is applied to the inertia m_1. The control force F is generated via an electromotor (actuator). As such, *the voltage* applied to the electromotor is denoted by the *control effort* in the ECP software. This control effort signal will be denoted by u and is measured in *Volts*.

As indicated in Figure 1, the position of m_1 or m_2 is indicated by x_1 or x_2 and is going is measured in *counts* by the encoder. For modeling purposes, both positions/rotations x_1 or

x_2 of the masses or inertias m_1 and m_2 may be used for estimating model parameters and validation of the 2DOF model. However, during the control design you must choose which position/rotation x_1 or x_2 will be used for feedback control. With the Force or Torque F acting on mass 1, the use of x_1 as a control output leads to a so-called co-located control system, where actuation and measurement is done at the same location (mass). With the Force or Torque F on mass 1, the use of x_2 as a control output leads to a so-called non co-located control system, where actuation and measurement is done at different locations. Summarizing:

> **The control effort u is the voltage applied to the electromotor to generate a force or momentum for the 2DOF system. The control effort signal u is measured in Volts.**
>
> **Either the position x_1 or x_2 can be used for feedback control design and implementation. The position signal x_1 or x_2 is measured in counts by the encoder.**

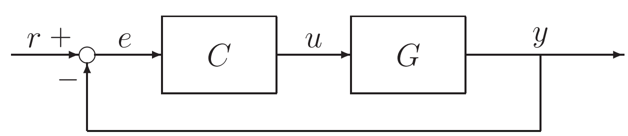

Figure 2: Block diagram of control system: G is mechanical system with input u and output $y = x_1$ or $y = x_2$, C is feedback controller with input $e := r - y$ and output u.

The control effort signal u can be considered as *an input signal* to the mechanical 2DOF system, while $y = x_1$ or x_2 denotes *an output signal*, as indicated in Figure 2. The feedback controller C that needs to be designed compares the position $y = x_1$ or x_2 with a desired position r via $e = r - y$ and should change the input u to control the mechanical system G to the desired position r.

First a model G of the dynamical behavior between u and $y = x_1$ or x_2 must be developed and validated. Then we can use that model for the design of a feedback controller C to control the vibrations in the position $y = x_1$ or x_2 of the mechanical system.

2.1.2 Dynamic model for 2DOF system

The derivation of the equations of motion for the 2DOF system can be found in the lecture notes on the control experiment. By approximating the dynamics of the electromotor via a simple conversion $F = K_p u$, the derivation of the equations of motion along with a Laplace transform, results in a transfer function model $G(s)$ with

$$y(s) = G(s)u(s)$$

that relates the Laplace transformed input force $u(s)$ and output signal $y(s) = x_1(s)$ or $x_2(s)$. In case the output is chosen as $y(s) = x_1(s)$, the transfer function $G(s)$ is given by

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{1}$$

whereas for the output $y(s) = x_2(s)$, the transfer function $G(s)$ is given by

$$G(s) = \frac{b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (2)$$

where the coefficients $b_i,\ i = 0, 1, 2$ and $a_j,\ j = 0, \ldots 4$ are determined by

$$\begin{array}{rcl} b_2 &=& m_2 \\ b_1 &=& d_2 \\ b_0 &=& k_2 \\ a_4 &=& m_1 m_2 \\ a_3 &=& (m_1 d_2 + m_2 d_1) \\ a_2 &=& (k_2 m_1 + (k_1 + k_2) m_2 + d_1 d_2) \\ a_1 &=& ((k_1 + k_2) d_2 + k_2 d_1 \\ a_0 &=& k_1 k_2 \end{array} \quad (3)$$

and include the mass m_1, m_2, damping d_1, d_2 and stiffness k_1, k_2 parameters.

To provide consistency with the signals used in the `ECP` software, it should be noted that the transfer functions $G(s)$ in (1) and (2) are used to relate the **control effort** $u(s)$, measured in Volts, to the position $y(s) = x_1(s)$ or $y(s) = x_2(s)$, measured in encoder counts. The presence of external hardware (electromotor, gear ratio, amplifier) and scaling of the signals u (in Volts) and the chosen encoder output y (in counts) are incorporated in the transfer function model $G(s)$.

2.1.3 Comparison of rectilinear and torsional system

The equations of motion and the resulting transfer function model $G(s)$ in (1) or (2) with parameters in (3) for the rectilinear system are similar to the torsional system, due to the mechanical similarity of both systems. In Figure 1, m_1, m_2 represent the mass or inertia, while x_1 and x_2 represent displacement or rotation angle respectively for the rectilinear 2DOF system and the torsional 2DOF system. The other coefficients d_1, d_2 and k_1, k_2 represent damping and stiffness and F denotes the control moment. In order to compare the torsional and the rectilinear system, consider the following remarks/questions and address them in your report:

- In case u is considered as *an input variable* and x_1 or x_2 as *an output variable*, what is the location and nature of these signals in the rectilinear and torsional system?
- Draw a schematic diagram of the torsional system using the same elements (carts with mass, springs and dampers) used in the diagram of the rectilinear system. From this diagram, what can you conclude on the (mechanical) differences between the torsional and the rectilinear system?
- What is the stiffness constant k_1 for the torsional system? Do you need to determine/estimate k_1?
- How does the transfer function $G(s)$ in (1) or (2) simplify for the torsional system? How does the transfer function $G(s)$ simplify if you assume there is no damping in the 2DOF system?

2.2 Dynamic response of second order systems

The coefficients in the transfer function model $G(s)$ are determined by the mass or inertia m_1, m_2, the stiffness k_1, k_2 and the damping d_1, d_2 of the mechanical system. A possible approach to estimate the unknown parameters of the 2DOF system, is to convert the 2DOF system in a 1DOF (single mass, damping and spring) second order system and estimate mass, damping and spring coefficients from a series of simple step response experiments. In order to understand how to obtain the model parameters from the step response, we first need to write down the actual step response as a function of the (unknown) model parameters for a 1DOF (single mass, damping and spring) second order system.

A mechanical system with a single mass m, damping d and stiffness k is given by the transfer function

$$y(s) = G(s)u(s),\ G(s) = \frac{1}{ms^2 + ds + k} = C\frac{\omega_n^2}{s^2 + 2\beta\omega_n s + \omega_n^2} \tag{4}$$

where

$$\begin{aligned} C &= \frac{1}{k} && \text{(steady state gain)} \\ \omega_n &= \sqrt{\frac{k}{m}} && \text{(undamped resonance frequency)} \\ \beta &= \frac{d}{2\sqrt{km}} && \text{(damping coefficient)} \end{aligned}$$

Applying a step $u(t) = U,\ t \geq 0$ of size U results in the output response

$$y(t) = CU\left[1 - \mathrm{e}^{-\beta\omega_n t}\sin(\omega_d t + \phi)\right] \tag{5}$$

where

$$\begin{aligned} \omega_d &= \omega_n\sqrt{1-\beta^2} && \text{damped resonance frequency} \\ \phi &= \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta} && \text{phase shift of response} \end{aligned}$$

Since the output response in (5) is an exponential decaying sinusoidal function, the parameters C, ω_d and the product $\beta\omega_n$ that determines the exponential decay can be determined from this step response.

3 Laboratory Experiments

3.1 Estimation of model parameters

As indicated in the previous section, a possible approach to estimate the unknown parameters of the 2DOF system, is to convert the 2DOF system in a 1DOF (single mass, damping and spring) second order system and estimate mass, damping and spring coefficients from a series of simple step response experiments.

To find the parameters of a particular (second order) transfer function of a 1DOF (single mass, damping and spring) second order system, consider a transfer function of a standard

2nd order system

$$G(s) = \frac{C\omega_n^2}{s^2 + 2\beta\omega_n s + \omega_n^2}$$

similar as in (4) where C denotes a gain, ω_n indicates the (undamped) resonance frequency and $0 \leq \beta < 1$ is a damping ratio that models (possible) damping of the 1DOF mechanical system. With this transfer function, a Voltage step input $u(t)$ of size U will results in the analytic solution given in (5).

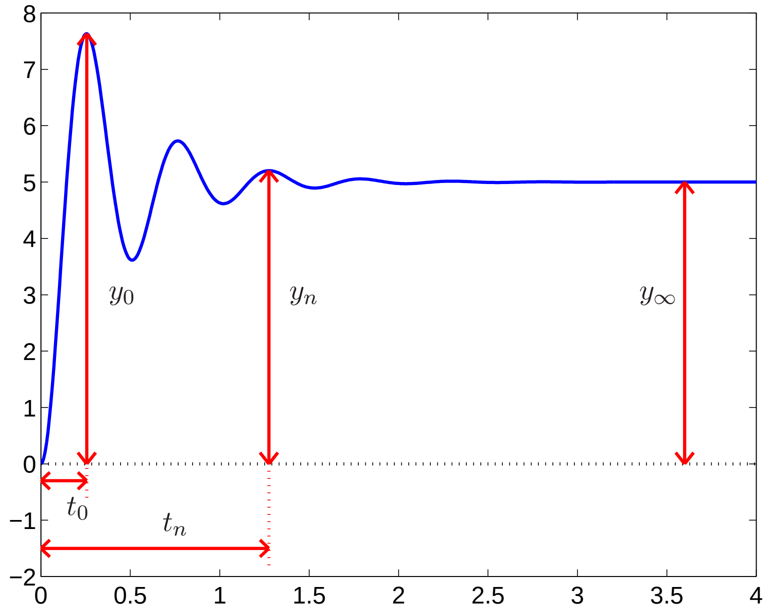

Figure 3: Step response of lightly damped 2nd order (1DOF) system

Using the analytic solution given in (5) and the typical step response of a lightly damped second order system indicated in Figure 3, it can be seen that ω_d and the product $\beta\omega_n$ can be estimated via

$$\begin{aligned} \hat{\omega}_d &= 2\pi\frac{n}{t_n - t_0} \\ \widehat{\beta\omega_n} &= \frac{1}{t_n - t_0}\ln\left(\frac{y_0 - y_\infty}{y_n - y_\infty}\right) \end{aligned}$$

and as a result, ω_n, β and C in (4) can be estimated by

$$\begin{aligned} \hat{\omega}_n &= \sqrt{\hat{\omega}_d^2 + (\widehat{\beta\omega_n})^2} \\ \beta &= \frac{\widehat{\beta\omega_n}}{\hat{\omega}_n} \\ \hat{C} &= \frac{y_\infty}{U} \end{aligned}$$

With the estimated values for C, ω_n and β, the values of the mass m, damping d and stiffness k in (4) (up to a scaling constant) can now be estimated via:

$$\begin{aligned}
\hat{k} &= \frac{1}{\hat{C}} \quad \text{(stiffness constant)} \\
\hat{m} &= \frac{1}{\hat{C}\,\hat{\omega}_n^2} \quad \text{(mass/inertia)} \\
\hat{d} &= \frac{2\hat{\beta}}{\hat{C}\,\hat{\omega}_n} \quad \text{(damping constant)}
\end{aligned}$$

When conducting your (step response) experiments in the lab to find the model parameters unknown model parameters (mass m_1, m_2, the stiffness k_1, k_2 and the damping d_1, d_2 parameters of the 2DOF mechanical system) keep the following items in mind:

1. How can you convert the 2DOF mechanical system in (several) 1DOF systems from which you can estimate the model parameters?

2. Propose experiments on different 1DOF systems that allow you to estimate all unknown parameters in the 2DOF system via dynamic experiments. List these experiments in your report.

3. Perform (step response) experiments and the estimation of the unknown model parameters several times (at least 5 times) to allow for statistical analysis. Mean value, standard deviation and confidence intervals (based on t-distribution) will have to be part of your report.

4. One has to realize the transfer functions given in (1), (2) or (4) relates torque (measured in Nm) to angular position (measured in rad/s). Since we apply torque via Voltage applied to a DC-motor and measure angular position in encoder counts, the transfer function will have a stiffness k (or similarly a gain $C = 1/k$) with the units Voltage/encoder counts.

5. Instead of estimating all model parameters of your 2DOF (mass m_1, m_2, the stiffness k_1, k_2 and the damping d_1, d_2) first and then validate if your complete 2DOF model with its 6 parameters is correct, try to validate the 3 parameters (single mass m, stiffness k and damping d) of the different 1DOF independently for each experiment you proposed under item 2. When there is an error in the estimation of one of the model parameters, it will be easier to detect from the individual validation steps of the 1DOF models. Validation of a 1DOF model can be done by setting the parameters `m1`, `d1` and `k1` in the `parameters.m` provided to you and running the `maelab.m` script file for a 1DOF model.

3.2 Validation of model

In order to validate the parameters of the individual 1DOF models that you are estimating and the 2DOF model of the mechanical system as a whole, one should compare measured data with simulation data generated by your model. Validation of the model is crucial before continuing with the design of a controller and can be done by editing the `parameters.m` file and running the `maelab.m` script file available on the computer in the laboratory.

Keep in mind that you will use a transfer function model $G(s)$ to design your controller. With an invalid or invalidated model, the controller cannot be designed reliably. Consider the following items for the validation of your model:

1. Propose experiments and the signals to be measured to validate your 1DOF models and the 2DOF model that you are constructing.

2. Which signals do you need to save, in order to be able to simulate data with your model and to compare simulated data with measured data? Think about *input* and *output* signals.

3. For each set of parameters (mass m, damping d and stiffness k) that you have estimated via a step response experiment for an individual 1DOF system, create in one figure the plot of a simulation and the measurement with the `maelab.m` script file to validate the parameters that you estimated. Include most relevant figures in your report.

4. Comment on how good the model is able to simulate the measured data. Give suggestions in case you need to adjust the model parameters in your report.

5. For the complete 2DOF system, inspect the Bode response of the model using the `maelab.m` script file. The peak values in the (amplitude) Bode response gives insight in the *resonance frequencies* and *resonance modes* of the system. Validate these resonance frequencies by applying a (small) sinusoidal excitation signal of the correct frequency to your 2DOF system. Since it is a resonance frequency, your measured output should have a maximum amplitude at this specific resonance frequency. Validate this by making the frequency slightly smaller or larger around the expected (modeled) resonance frequency.

In order to compare a measurement with a simulation, you have to save your measured data of your relevant dynamic experiments in a text file (extension `.txt`) via the `export raw data` option in the `ECP` software. Do not start the filename with a number and make sure you save your data under your working directory! Subsequently you have to prepare the text file of the saved data so that it can be read it into Matlab. This is done by the following editing steps:

1. First line in text file: Comment out the first line with a `%`

2. Second line in text file: Enter `dummy=` before the opening bracket `[`

3. Last line in text file: Put a semicolon `;` behind the closing bracket `]`

4. After last line in text file: define time t, input u and output y by selecting the appropriate columns from the `dummy` variable. For example, if you have selected to save the control effort (input u) and the encoder 1 position (outpu y), this can be done by adding the following lines to the end of the text file:

```
t=dummy(:,2);
y=dummy(:,3);
u=dummy(:,4);
clear dummy;
```

5. Save the raw text file as a file with the extension `.m`

The result is a Matlab script file that can be run from Matlab. When you run the file, the time vector `t`, the input signal `u` and the output signal `y` are available in the Matlab workspace for plotting and comparison purposes to validate your model. The script file `maelab.m` can also directly read the data files for comparison purposes and the editing steps above are only necessary if you like to plot the data individually in Matlab to create figures for your report.

4 Design of Feedback Control for 2DOF System

4.1 Control specifications

In the `ECP` software, a (continuous time) PID controller of the form

$$C(s) = k_p + \frac{k_i}{s} + k_d s \tag{6}$$

can be implemented on the mechanical system. The design variables of the controller are k_p (proportional gain), k_i (integral gain) and k_d (differential gain). The design variables in (6) have to be used to design a feedback controller such that the controlled mechanical system satisfies the following conditions. The controller $C(s)$ should be able to

1. **Choose to use either the position $y = x_1$ of mass/inertia m_1 or the position $y = x_2$ of mass/inertia m_2 for feedback purposes.**

2. **Based on the model $G(s)$ in (1) for $y = x_1$ or (2) for $y = x_2$, design a controller $C(s)$ such that:**

    - **a stable feedback control system is obtained,**
    - **the position of the $y = x_1$ or $y = x_2$ used for feedback purposes can be changed as fast as possible with not more than 25% overshoot.**

With the knowledge of a validated model $G(s)$, the feedback controller $C(s)$ can be designed. For that purpose, the Matlab script file `maelab.m` can be used. With this script file you can (interactively) enter values for the control design parameters k_p, k_i and k_d to design a controller that satisfies the control specifications. To design a controller $C(s)$ that will satisfy the above mentioned three requirements, first a P-controller is designed, Subsequently, the P-controller is extended to a PD- and even PID-controller to match all requirements.

4.2 Design of P-controller

For the design of the P-controller, heuristic tuning rules such as Ziegler and Nichols can be used. Alternatively, with the knowledge of the model $G(s)$, a stabilizing proportional controller can be found via the root-locus method or with the frequency domain method. Irrespective of the method you use to design the P-controller, address the following items in your report:

- Explain the method you use to design a P-controller and how you evaluate the stability of your closed-loop system.
- When you have implemented a stable feedback system with the P-controller, increase the value of the proportional gain k_p in small steps. Explain why the response of the mechanical system becomes faster, but less damped.

4.3 Design of PD- & PID-controller

Using only a P-controller gives a poorly damped mechanical system and in order to improve the overshoot and damping of the controlled system, a PD-controller is used. With the knowledge of the model $G(s)$, a stabilizing PD-controller can be found via the root-locus method or with the frequency domain method. Using such a model-based control design approach, address the following items in your report:

- Explain the method you use to design a PD-controller and how you evaluate the stability of your closed-loop system.
- Give a well motivated choice for the values of k_p and k_d of your PD-controller. Explain the improvements in damping and reduction of overshoot you have obtained by the use of a PD-controller.
- Is the PD-controller able to position $y = x_1$ of mass/inertia m_1 or the position $y = x_2$ of mass/inertia m_2 without any steady state error? With respect to this question, compare also the behavior of the torsional and the rectilinear system with a PD-controller. Can you give an explanation?

In case a steady state error is still present in your closed-loop step response, an additional integral action in the controller is able to reduce steady state errors. For that purpose, the PD-controller is extended to a PID-controller. For the design of a PID-controller give a well motivated choice for the values of k_p, k_d and k_i of your PID-controller in your report. Explain the reduction in steady-state error by the use of a PID-controller.

4.4 Sensitivity analysis

Once you have designed a properly working feedback controller, you should check the sensitivity of the feedback system to any chances in the system or controller parameters. The system parameters can be changed by changing spring stiffness or mass/inertia properties. Try to reduce or increase the mass/inertia of the system, while using the same feedback controller. For that purpose, turn of the control, change the mass/inertia and turn the control back on. Be careful, **the feedback system might become unstable** and in that case you should turn of the control as soon as possible.

The controller parameters can be changed by changing the k_p, k_i or k_d with approximately 10%. Again, **the feedback system might become unstable** and in that case you should turn of the control as soon as possible. Address the following items in your report:

- How sensitive is your designed feedback system to system parameter changes?
- How sensitive is your designed feedback system to control parameter variations?
- Do you have an explanation for the sensitivity to the various parameters? Comment on the quality requirements of the model.

5 Laboratory report

By the end of the position control experiment you should have estimated (with error analysis) and validated the parameters of a dynamic model of both the rectilinear and torsional system. You should also have a stabilizing feedback controllers for both the rectilinear and torsional system designed on the basis of the dynamic models of the system.

The results of your laboratory work have to be reported in a coherent written report. The report will be graded on the layout and the contents that should (at least) contain the following items:

- Estimation and error analysis of parameters in the model of the 2DOF system.
- Comparison of dynamic behavior of rectilinear and torsional system.
- Comparing dynamic experiments with dynamic model simulations.
- Design procedure for P, PD, PID (or state feedback) controller on the basis of a dynamic model of the system to be controlled
- Implementation of feedback controllers and comparing closed-loop dynamic experiments with closed-loop dynamic model simulations
- Sensitivity (robustness) analysis by (closed-loop) experiments that examine settling time and overshoot variations due to changes in mass/inertia parameter variations

In addition, refer to the questions posed throughout this laboratory handout. Make sure to include your answers to these questions in the appropriate sections of your laboratory report.

— end of laboratory handout —