MAE 119: Introduction to Renewable Energy
Winter 2013
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Quiz 1: Population, Human quality of life, Access to energy, and Implications.

Open Book/Open Notes (but you shouldn't need them).
All parts equally weighted.

a. The population growth rate, r, is defined as $r(t) = \frac{1}{P(t)}\frac{dP(t)}{dt}$ and in general can change in time so that r=r(t). Here the population given as P(t). Find P(t) with $P(t=0)=P_0$. Hint: $\int_a^b \frac{dP}{P} = \ln P\Big|_a^b$

b. Suppose that we now live in a world where the annual per capita energy access, E, is fixed at a constant value. Given the correlations between energy access and population growth rate that we have discussed in class, what can you say about the population growth rate, r(t)?

c. Let us model the dependence of population growth rate on energy access with the expression $r(t) = r_0 \frac{E_0}{E(t)}$ where $r_0 = r(E = E_0) \sim const$ is the initial growth rate when E is equal to the reference value, i.e. $E=E_0$ which we take to occur at a reference time t=0. If per-capita energy access is growing such that $E(t) = E_0 \exp[r_0 t]$, find the population vs. time P(t). Compared to the answer in part (c), which scenario has a larger population for late time $t \gg 1/r_0$?

d. Using your knowledge of the HDI index vs per capita energy access, sketch the HDI vs time for the two scenarios found in (b) and (c). Which of these two scenarios is closer to current conditions in the world?

Quiz 1 - MAE119 W2013 SOLUTION

a) $r(t) = \frac{1}{P(t)} \frac{dP}{dt} \rightarrow \frac{dP(t)}{P(t)} = r(t)\,dt$

$$\int_{t_0}^{t} \frac{dP}{P} = \int_{t_0}^{t} r(t)\,dt \longrightarrow \ln P\Big|_{t_0}^{t} = \int_{t_0}^{t} r(t)\,dt$$

3 pts

$$p(t) = P_0 \exp\left[\int_{t_0}^{t} r(t)\,dt\right] \text{ if } r = r(t).$$

-2 pts

b) If $r = r_0 \sim$ const then

$E =$ const $\nearrow$

$$P(t) = P_0 e^{r_0(t-t_0)}$$

5 pts

c) $r = r_0 \frac{E_0}{E}$; $E = E_0 e^{r_0(t-t_0)}$

$$\therefore\ r = r_0 e^{-r_0(t-t_0)}$$

$$\therefore\ P(t) = P_0 \exp\left[\int_{t_0}^{t} r_0 e^{-r_0 t}\,dt\right]$$

3pts

define

$$I \equiv \int_{t_0}^{t} r_0 e^{-r_0(t-t_0)} dt$$

$$= -e^{-r_0(t-t_0)}\Big|_{t_0}^{t}$$

$$= 1-e^{-r_0(t-t_0)}$$

then

$$P(t) = P_0 \exp\left[1-e^{-r_0(t-t_0)}\right] \quad c$$

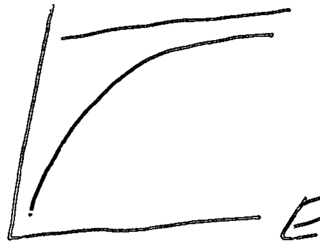
if $t \to t_0$ $P \to P_0$ ✓

if $t \to \infty$ $P \to P_0 \exp(1)$ i.e.

P asymptotically approaches a finite value while in (b) P diverges exponentially

d) We know HDI:

5



Thus $HDI \to 1$ late in time while $HDI = \text{const}$ in (b)