**MAE 119 Professor G.R. Tynan**
**Winter 2018**
**Homework 4**

**Assigned 6 Februrary 2018 Due 19 February 2018**

1. Wave Power:

Using internet resources, determine the typical wave amplitude and wavelength for ocean waves off of the eastern and western coasts of the US. Also examine the Gulf of Mexico region. Use these values to estimate the power per unit length along the wavefront (i.e. parallel to the coastline) for these regions, and then estimate the maximum power that could be converted into electricity if 10% of the available coastline were dedicated to this purpose. What fraction of total US electrical power demand does this represent?

2. Wave power: Find the power production from a wave machine that reduces the wave height by a factor of 2. In other words, if the incident waves have a height a. The machine then only extracts a portion of the available wave energy, allowing a wave of height a/2 to exit the machine. If a=1 m, and the period is 10 seconds, estimate how much power could be produced from a machine 100m long in the direction parallel to the wave face.

3. Hot dry rock Geothermal Energy:

A geothermal heat mine located at a depth between 10 and 11 km deep, and the volume to be mined has horizontal dimensions of 1 km x 1 km. Use the MIT Report on Hot Rock Geothermal Energy (located on the class website) to estimate the energy required to (a) drill and (b) operate such a heat mine. Cite your sources. (c) if the mine has a power output of 100 MW, on what time-scale does the rock cool down? How might this relate to the lifetime of the mine?

4. Solar Thermal Power:

During the lecture in class about solar thermal power systems, we found the following time-dependent energy balance equation for the case where there was no heat input into the system (e.g. night time without any auxiliary power input from e.g. natural gas combustion).

$$\rho C_p V \frac{\partial T}{\partial t} = -\frac{1}{\eta_{th}} P_{out}$$

a. Define and explain the meaning of each term in this equation.
b. Assuming all terms except T are constant and the temperature is a function of time, find the exact solution T=T(t).
c. If the power output of such a system is 50 MW, the working fluid was molten salt (see e.g. http://en.wikipedia.org/wiki/Solar_thermal_energy for more information the properties of such a thermal storage system.) estimate the volume V such that the e-folding time of the system is at least 12 hours (i.e. enough to last through the night).

5. Solar Thermal Power

For the system described in the previous problem:

a) If there ***is*** power input into the system from the sun, which we denote as $P_{in}$ what would be the new energy balance equation? In order words, how would you modify the equation above to account for the power input?
b) If this input power from the sun is constant in time, and the system had zero output power, what would the functional form of T(t) be?
c) If the solar intensity is 300 $W/m^2$ and the thermal conversion efficiency is 30%, what is the area of the collecting mirrors required for a 100 MW output power? You may assume that the plant would be operating in steady-state.

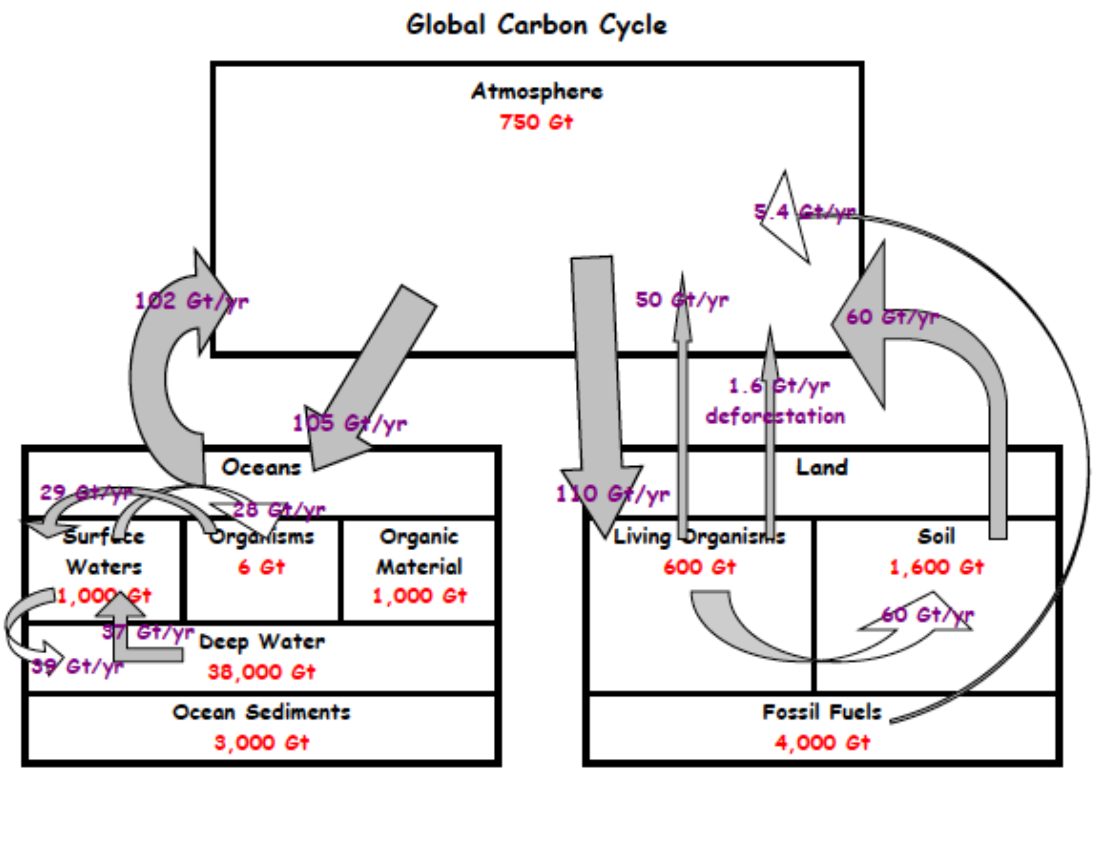
6. Tidal Power

The Rance Tidal Power Station is a utility-scale tidal power system located in France. Find the power output of this station. What is the typical height of the tides in this system? What is the tidal basin area (i.e. the surface area of the water impounded and utilized in this station? Using the simple models developed in lecture, compare the idealized power output to the published power output of this station. What fraction of France's electricity demand is met by this station? Using internet resources, what is the estimated global power capacity of feasible tidal basins that could be used to generate electrical power? What fraction of current global demand does this represent?

7. Carbon Balance:

A more complete control-volume model of the Earth's C-balance for the conditions of the atmosphere in ~2005 is given in the figure below. Suppose the C transfer rates between atmosphere, ocean, and land given the figure were the same during pre-industrial times, and that during pre-industrial times, de-forestation was negligible and the atmosphere held 600 Gt of C in the form of $CO_2$.

a) Referring to our discussion of the Earth's C-balance, estimate the net effective equilibration time for atmospheric C.
b) If this equilibration time find in part (a) doesn't change and CO2 emission rates were to stop growing and were to be held at today's value of ~10 Gt-C/year, what will be the steady-state atmospheric C content?
c) Using our simple IR radiation transport model developed in class, what would the new equilibrium IR transmission coefficient be?
d) Using the other values for the Earth's heat balance model given in lecture notes, estimate the change in temperature for the atmosphere and Earth's surface.



8.

*Solar Thermal:* A solar power tower design concept has a centrally located point-like target illuminated by an array of heliostat mirrors that can be oriented to reflect sunlight onto the target. The ratio of the mirror collecting area to the target area is a factor of 1000.

*a.* If the direct normal incidence (DNI) solar irradiation is 1000 $W/m^2$, what is the incident heat flux to the target?

*b.* If the working fluid of the power plant removes heat from the target at a rate of 500 $kW/m^2$, what will be the equilibrium temperature of the target? [Hint: write a power balance for a unit surface area of the target and then recall that the emitted heat flux from a radiating body goes like $\sigma T^4$ where $\sigma$ denotes the Stefan-Boltzmann constant which has a value of ~6x10-8 $W/m^2\text{-}K^4$]

*c.* If the working fluid has a temperature that is half of the target temperature, estimate the thermal conversion efficiency of an ideal heat engine deployed in this system.

*d.* Suppose a cloud layer moves over that has a thickness of 1km. The cloud is composed of aerosol particles with a cross-sectional area of $10^{-9}$ $m^2$. These particles have a density of $10^6$ particles/$m^3$. What is the DNI now? If the plant were to keep operating, by how much will the power plant power output decline (you may neglect any change in the thermal conversion efficiency).

Solution to Homework 4 MAE 119 W2018

## Problem 1 – Wave Power

The wave power per unit length is given as

$$P = \frac{1}{4\sqrt{2\pi}}\rho g^{3/2}\lambda^{1/2}a^2,$$

where $\rho = 10^3\,\mathrm{kg/m^3}$, $g = 9.8\,\mathrm{m/s^2}$, and $\lambda = \frac{2\pi g}{\omega^2} = \frac{gT^2}{2\pi}$. Using data from Coastal Data Information Program,[1] we can get the following table 1,

| Region | $a$ (m) | $T$ (s) | $P$ ($\mathrm{kW\,m^{-1}}$) |
|---|---|---|---|
| West Coast | 1.12 | 7.75 | 37.6 |
| East Coast | 0.47 | 5.70 | 4.75 |
| Gulf Coast | 0.49 | 4.20 | 3.77 |

Table 1: Estimations of wave power per unit length in different regions of United States

West coast has a length of $2\times10^6$ m (exclude Alaska and Hawaii), east coast $3\times10^6$ m and Gulf coast $2.6\times10^6$ m. Then total electricity power from waves along the coastlines is about

$$P_{\mathrm{tot}} = 10\% \times \left(37.6\times2\times10^6 + 4.75\times3\times10^6 + 3.77\times2.6\times10^6\right) \approx 10\,\mathrm{GW}.$$

US electricity demand is roughly 4000 TWh per year, the required power is about

$$P_{\mathrm{req}} = \frac{4000}{365\times24} \approx 460\,\mathrm{GW}.$$

Obviously, the total wave power only accounts for 2% of the electricity demand of US.

[1] http://cdip.ucsd.edu/?nav=historic&sub=data&units

Problem 2 – Wave Power

Wave power per unit length along the wave face entering the machine is

$$P_{in} = \frac{1}{4\sqrt{2\pi}} \rho g^{3/2} \lambda^{1/2} a_{in}^2$$

where $a_{in}$ denotes the wave height entering the machine, and the wave power per unit length exiting the machine is given as

$$P_{out} = \frac{1}{4\sqrt{2\pi}} \rho g^{3/2} \lambda^{1/2} a_{out}^2$$

the net power output, using the first law of thermodynamics and assuming perfect conversion efficiency is given as

$P_{net} = P_{in} - P_{out.}$

One can easily input values, and account for 100 m long machine to find the net power output.

Problem 3 – Geothermal Power

## a) Drilling Cost

According to Figure 6.1 in MIT Report on Hot Rock Geothermal Energy, the estimated cost to drill a well with a depth of 10 km is about 20 million dollars per year. In addition, the report notes that each well has roughly 5 $km^3$, which means we have 2 wells as the total volume is 10 $km^3$. The average price of electricity for industrial sector in 2016 is about 6.6 cent/kWh.[1] The energy cost per year to drill the mine (2 wells) is about $2 \times \frac{20\times 10^6}{6.6\times 10^{-2}} = 6\times 10^8$ kWh = 0.6 TWh.

## b) Operating Cost

The Operating and maintenance costs range from 0.01 to 0.03 dollar per kWh. Most geothermal power plants can run at greater than 90% availability.[2] For a 100 MW power plant, the operating cost per year is about $0.9 \times 0.02 \times 100 \times 10^3 \times 365 \times 24 = 15.8$ (M$).

## c)

The characteristic time scale is

$$
\begin{aligned}
\tau &= \frac{\rho C_p V}{S_{\text{out}}^{\text{tot}}} \frac{((1-\eta_{\min})T(d) - T_{\text{surf}})}{(1-\eta_{\min})}, \\
&= \frac{3\times 10^3 \times 10^3 \times 3.3\times 10^3 \times 3.3\times 10^3 \times 10^3}{100\times 10^6} \frac{((1-0.2)500 - 300)}{(1-0.2)}, \\
&\approx 1200\,\text{yr},
\end{aligned}
$$

where $\eta_{\min} = 0.5\times(1-\frac{300}{500}) = 0.2$ is the minimum efficiency. This time scale represents the lifetime of the mine, that is after this period it is no longer economically feasible to extract energy from the rocks, since the efficiency will decrease below $\eta_{\min}$ and the cost of extracting any more energy would become prohibitively expensive.

[1] http://www.eia.gov/electricity/monthly/epm_table_grapher.cfm?t=epmt_es1a
[2] https://energy.gov/eere/geothermal/geothermal-faqs

4. Solar Thermal Power

**a)**

- $\rho$ – fluid mass density
- $C_p$ – fluid specific heat at constant pressure
- $V$ – volume of storage tank incorporated into solar thermal system
- $T$ – fluid temperature
- $P_{\mathrm{out}}$ – output power of the system
- $\eta_{\mathrm{th}}$ – thermal conversion efficiency

**b)**

Since all terms except $T$ are constant, the equation is reduced to

$$\frac{\partial T}{\partial t} = -\frac{P_{\mathrm{out}}}{\rho C_p V \eta_{\mathrm{th}}}.$$

Upon integration from 0 to $t$ we can obtain

$$\int_{T_0}^{T(t)} dT = -\frac{P_{\mathrm{out}}}{\rho C_p V \eta_{\mathrm{th}}} \int_0^t dt,$$

$$T(t) - T_0 = -\frac{P_{\mathrm{out}}}{\rho C_p V \eta_{\mathrm{th}}} t.$$

**c)**

Assume that the average density of molten salt is $\rho = 1850\,\mathrm{kg/m^3}$ and specific heat is $C_p = 1.5\,\mathrm{kJ/kg/K}$. The volume $V$ can be represented as

$$\begin{aligned} V &= -\frac{P_{\mathrm{out}}}{\rho C_p V \eta_{\mathrm{th}}} \frac{\Delta t}{\Delta T}, \\ &= -\frac{50 \times 10^6\,\mathrm{J/s}}{1850\,\mathrm{kg/m^3} \times 1500\,\mathrm{J/kg/K} \times 0.3} \frac{12 \times 3600\,\mathrm{s}}{-100\,\mathrm{K}}, \\ &= 2.6 \times 10^4\,\mathrm{m^3}. \end{aligned}$$

Problem 5 – Solar Thermal Power

**a)**

If there is a power input to the system then the equation becomes

$$\rho C_p V \frac{\partial T}{\partial t} = P_{\text{in}} - \frac{P_{\text{out}}}{\eta_{\text{th}}}.$$

**b)**

If this input power from the sun is constant in time, and the system has zero output power, the equation is reduced to

$$\rho C_p V \frac{\partial T}{\partial t} = P_{\text{in}}.$$

Here, $P_{\text{in}} = I_0 A$ denotes the solar radiation power input, where $I_0$ is the incident solar radiation intensity and $A$ is the collector area normal to the incident radiation. The solution for $T$ is then written as

$$T(t) - T_0 = \frac{P_{\text{in}}}{\rho C_p V} t.$$

**c)**

For steady-state operation, $\partial_t T = 0$. Therefore, the input power from solar radiation is balanced with output power of the system, i.e.

$$\begin{aligned} I_0 A &= \frac{P_{\text{out}}}{\eta_{\text{th}}}, \\ A &= \frac{P_{\text{out}}}{\eta_{\text{th}} I_0}, \\ &= \frac{100 \times 10^6\,\text{W}}{0.3 \times 300\,\text{W/m}^2}, \\ &= 1.11 \times 10^6\,\text{m}^2. \end{aligned}$$

Problem 6 – Tidal Power

Typical parameters are below:[1]

- Peak power output $P_{pk} = 240\,\mathrm{W/m^2}$
- Average height $h = 8\,\mathrm{m}$
- Tidal basin area $A = 2.23 \times 10^7\,\mathrm{m^2}$
- Capacity factor is 0.26
- Average power $P_a = 64\,\mathrm{MW}$

According to the model developed in class, the estimated power production is

$$\begin{aligned} P_{avg} &= \frac{1}{2}\frac{\rho g A h^2}{T} \times 0.26, \\ &= \frac{1}{2}\frac{1000\,\mathrm{kg/m^3} \times 10\,\mathrm{m/s^2} \times 2.23 \times 10^7\,\mathrm{m^2} \times 8\,\mathrm{m}}{6 \times 3600\,\mathrm{s}} \times 0.26, \\ &= 84\,\mathrm{MW}. \end{aligned}$$

The value is reasonably close to the actual real average power production $P_a = 64\,\mathrm{MW}$. This power station supplies 0.012% of France's power demand. Summing up the energy production capacity of major world tidal basin sites gives $P_{tot} = 580\,\mathrm{TWh/yr}$. This value is roughly 3% of the total world electricity consumption (23 000 TWh/yr).

[1] https://en.wikipedia.org/wiki/Rance_Tidal_Power_Station

7. Carbon balance.

a) Examining the figure in the problem statement giving the global carbon cycle, we see that the net C exchange from the atmosphere to the oceans is 105 Gt/yr – 102 Gt/yr = 3Gt/yr from atmosphere to ocean. Similarly the net flux from the atmosphere to the land is 110 Gt/yr-50-1.6-60 = -1.6 Gt/yr, i.e. a net addition of 1.6 Gt/yr from deforestation [ note that we don't count the 5.4 Gt/yr of C addition to the atmosphere due to fossil fuel consumption for the purposes of this calculation]. Thus the net loss rate of C from the atmosphere is 3 Gt/yr – 1.6 Gt/yr = 1.4 Gt/yr net loss from the atmosphere to the combined ocean and land. We then estimate the net effective equilibration time to be given as

$t_{net}$ ~ C mass in atmosphere / net rate of C loss from the atmosphere ~750 Gt/1.4Gt/yr. This gives $t_{net}$ ~ 500 years or so.

b) From our carbon balance model developed in class, we know that for a constant C injection rate, the change in the equilibrium C content in the atmosphere is given as $Q_C$ $t_{net}$. For the given values, we then have the change in C content to be 10 Gt/yr * 500 yr ~ 5000 Gt, i.e. about a factor of 6-7x higher than the initial value of C content (of 750 Gt).

c) IR transmission coefficient will then decrease by a factor of $e^{-6}$ to $e^{-7}$.

d)

8. Solar Thermal

a ) $P_{in}$=C $I_0$ = 1000 x 1000 W/m$^2$=10$^6$ W/m$^2$

b) Power balance equation reads

$$CI_0 = \sigma T^4 - P_{out}$$

solving for T yields

$$T = \sqrt[1/4]{\frac{CI_0 - P_{out}}{\sigma}}$$

substituting the given numerical values gives  T ~ 1700 K.

c) $\eta = 1 - \frac{T_c}{T_h} = 1 - \frac{300K}{850K} \approx 0.65$

d) $I(d) = I_0 \exp(-n\sigma d)$ ==> $I(d)/I_0 = \exp(-10^6 10^{-9} 10^3) = e^{-1}$

Thus DNI is reduced by a factor of 1/e ~ 0.36. Thus power output decline would be ~64%.