MAE108: Probability and Statistical Methods for Mechanical and Environmental Engineering
Midterm 1

To receive full credit you have to provide a detailed explanation of your answers.

1. (**20 points**) Four students in this class (whose names are A, B, C, and D) decided to follow their instructor's advice and prepare for a midterm together. They decided that two of them would work on a problem and then report their solution to the group. These two lucky students are to be selected at random by drawing names from a hat.

   (i) Write down a sample space of this selection. **(5 pts)**

   (ii) Assuming that all outcomes are equally likely (i.e., in the absence of cheating with the hat), what is the probability that student A will be chosen to work on the problem? **(15 pts)**

Solution:

(i) The sample space consists of six possible pairs of students: {AB, AC, AD, BC, BD, CD}

(ii) Student A appears in 3 out of 6 possible pairs. Hence the probability of her working on the problem is 3/6 = 0.5.

2. (**30 points**) An electrical circuit in Figure 1 consists of two components a and b. Since these components are connected in parallel, the circuit keeps working as long as at least one component remains operational. A reliability database suggests that the probability of component a being operational is 0.99, the probability of component b being operational is 0.98, and the probability of the two components being simultaneous operational is 0.9702.

   (i) What is the probability of finding the circuit operational? **(15 pts)**

   (ii) What is the probability that the two components fail simultaneously? **(15 pts)**

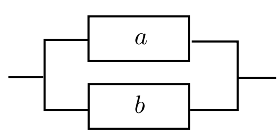

Figure 1: An electrical circuit consisting of two components a and b, which are connected in parallel.

Solution: Let A denote an event "component a is working"; its probability is $P(A) = 0.99$. Let B denote an event "component b is working"; its probability is $P(B) = 0.98$. The probability of the two components working at the same time is $P(A \cap B) = 0.9702$.

(i) The circuit is operational if either one or both components are operational. This corresponds to event $A \cup B$, whose probability is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.99 + 0.98 - 0.9702 = 0.9998.$$

(ii) Event "the two components fail simultaneously" is written as $\overline{A} \cap \overline{B}$. Its probability is

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.9998 = 0.0002.$$

3. (**50 points**) A full deck of playing cards consists of 52 standard cards.

   (i) Determine the probability of drawing a queen on the first try. **(10 pts)**

   (ii) Determine the probability of drawing two queens in two consecutive tries *with replacement*. The latter means that you put the card you draw in the first try back into the deck before drawing the second time. **(15 pts)**

   (iii) Determine the probability of drawing two queens in two consecutive tries *without replacement*. The latter means that the card from the first draw is removed from the deck. **(25 pts)**

   Hint 1: In each case, start by defining the event of interest (e.g., in all three questions let A denote an event of drawing a queen). Hint 2: Whenever necessary, determine whether two events are independent or not. Hint 3: You can report your answers in simple fractions.

Solution:

(i) Let A denote an event of drawing a queen. Since there are 4 queens among 52 cards, drawing any one of them has the probability $P(A) = 4/52 = 1/13$.

(ii) Let A denote an event of drawing a queen on the first try. Let B denote an event of drawing a queen on the second try. The event of interest, i.e., drawing two queens in two tries, is $A \cap B$. Since whichever card is drawn first is placed back into the deck, the second attempt is independent of the first, i.e., A and B are independent. Therefore, their probability is

$$P(A \cap B) = P(A)P(B) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}.$$

(iii) If A happened, the deck has 51 cards and contains 3 queens. Therefore, the *conditional* probability of drawing the second queen is $P(B|A) = 3/51$. Consequently,

$$P(A \cap B) = P(B|A)P(A) = \frac{3}{51} \times \frac{1}{13} = \frac{1}{17} \times \frac{1}{13} = \frac{1}{221}.$$