MAE108, Assignment 2

Problems 2.4, 2.6, 2.8, 2.10, 2.12, 2.14, 2.16, 2.18 from the textbook.

In essence, the concepts and tools developed in this chapter constitute the essential fundamentals necessary for correctly applying probability in engineering. In the ensuing chapters, particularly Chapters 3 and 4, additional analytical tools will be developed based on the fundamental concepts expounded in this chapter.

PROBLEMS

2.1 Suppose the travel time between two major cities A and B by air is 6 or 7 hr if the flight is nonstop; however, if there is one stop, the travel time would be 9, 10, or 11 hr. A nonstop flight between A and B would cost \$1200, whereas with one stop the cost is only \$550. Then, between cities B and C, all flights are nonstop requiring 2 or 3 hours at a cost of \$300. For a passenger wishing to travel from city A to city C,

(a) What is the possibility space or sample space of his travel times from A to B? From A to C?

(b) What is the sample space of his travel cost from A to C?

(c) If T = travel time from city A to city C, and S = cost of travel from A to C, what is the sample space of T and S?

2.2 The settlement of a bridge pier, say Pier 1, is estimated to be between 2 and 5 cm. Similarly, the settlement of an adjacent pier, Pier 2, is also estimated to be between 4 and 10 cm. There will, therefore, be a possibility of differential settlements between these two adjacent piers.

(a) What would be the sample space of this differential settlement?

(b) If the differential settlements in the above sample space are equally likely, what would be the probability that the differential settlement will be between 3 and 5 cm?

2.3 The direction of the prevailing wind at a particular building site is between due East ($\theta = 0^\circ$) and due North ($\theta = 90^\circ$). The wind speed V can be any value between 0 and ∞.

(a) Sketch the sample space for wind speed and direction.

(b) Denote the events:

$$E_1 = (V > 35 \text{ kph});$$
$$E_2 = (15 \text{ kph} < V \leq 45 \text{ kph})$$
$$E_3 = (\theta \leq 30^\circ)$$

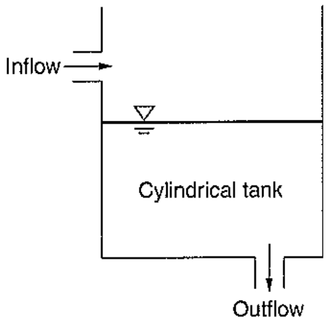
Identify the events E_1, E_2, E_3, and $\overline{E}_1$ within the sample space of Part (a).

(c) Use new sketches to identify the following events:

$$A = E_1 \cap E_3; \qquad B = E_1 \cup E_2; \qquad C = E_1 \cap E_2 \cap E_3$$

Are the events A and B mutually exclusive? How about between the events A and C?

2.4 A cylindrical tank is used to store water for a town as shown in the figure below. On any given day, the water supply inflow per day may fill the tank with an additional 6, 7, or 8 ft of water. The daily demand or consumption of the water for the town will draw down the water level in the tank by an amount equivalent to 5, 6, or 7 ft of the water in the tank.



Cylindrical water tank.

(a) What are the possible combinations of inflow and outflow of water in the tank in a given day?

(b) If the water level in the tank is 7 ft from the bottom at the start of a day, what are the possible water levels in the tank at the end of the day?

(c) If the amounts of inflow and outflow of water for the tank are both equally likely, what would be the probability that there would be at least 9 ft of water remaining in the tank at the end of the day?

2.5 A 20-ft cantilever beam is shown in the figure below. Load $W_1 = 200$ lb, or $W_2 = 500$ lb, or both may be applied at the midpoint B or at the end of the beam C. The bending moment induced at the fixed support A, M_A, will depend on the magnitudes of the loads at B and C.

20-ft cantilever beam.

(a) Determine the sample space of M_A.

(b) Define the following events:

$$E_1 = (M_A > 5{,}000 \text{ ft-lb})$$
$$E_2 = (1{,}000 \leq M_A < 12{,}000 \text{ ft-lb})$$
$$E_3 = (2{,}000, 7{,}000 \text{ ft-lb})$$

Are the events E_1 and E_2 mutually exclusive? Explain why.

(c) Assume the following respective probabilities for the positions of the two loads:

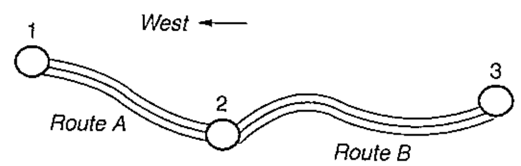
$$P(W_1 \text{ at B}) = 0.25$$
$$P(W_1 \text{ at C}) = 0.60$$
$$P(W_2 \text{ at B}) = 0.30$$
$$P(W_2 \text{ at C}) = 0.50$$

Assuming that the positions of W_1 and W_2 are statistically independent; what are the respective probabilities associated with each of the possible values of M_A?

(d) Determine the probabilities of the following events:

$$E_1,\ E_2,\ E_3,\ E_1 \cap E_2,\ E_1 \cup E_2, \text{ and } \overline{E}_2$$

2.6 Two cities 1 and 2 are connected by route A, and route B connects cities 2 and 3 as shown in the figure below. Let us denote the eastbound lanes as A_1 and B_1, and the westbound lanes as A_2 and B_2.



Routes connecting three cities.

Suppose the probability is 95% that one of the two lanes in route A will not require major resurfacing of the pavement for at least 2 years; the corresponding probability for a lane in route B is only 85%.

(a) Determine the probability that route A will require major resurfacing in the next 2 years. Do the same for route B. Assume that if one lane of a route needs major resurfacing, the chance that the other lane of the same route will also need resurfacing is 3 times its original probability.

(b) Assuming that the need for resurfacing in routes A and B are statistically independent, what is the probability that the road between cities 1 and 3 will require major resurfacing in two years?

2.7 From past experience, it is known that, on the average, 10% of welds performed by a particular welder are defective. If this welder is required to do three welds in a day,

(a) What is the probability that none of the welds will be defective?

(b) What is the probability that exactly two of the welds will be defective?

(c) What is the probability that all the welds for a day are defective?

It is assumed that the condition of each weld is independent of the conditions of the other welds.

2.8 On a given day, casting of concrete structural elements at a construction project depends on the availability of material. The required material may be produced at the job site or delivered from a premixed concrete supplier. However, it is not always certain that these sources of material will be available. Furthermore, whenever it rains at the site, casting cannot be performed. On a given day, define the following events:

E_1 = there will be no rain

E_2 = production of concrete material at the job site is feasible

E_3 = supply of premixed concrete is available

with the following respective probabilities:

$$P(E_1) = 0.8; \qquad P(E_2) = 0.7; \qquad P(E_3) = 0.95;$$
$$\text{and} \qquad P(E_3 | \overline{E}_2) = 0.6$$

whereas E_2 and E_3 are statistically independent of E_1.

(a) Identify the following events in terms of E_1, E_2, and E_3:

(i) A = casting of concrete elements can be performed on a given day.

(ii) B = casting of concrete elements cannot be performed on a given day.

(b) Determine the probability of the event B.

(c) If production of concrete material at the job site is not feasible, what is the probability that casting of concrete elements can still be performed on a given day?

2.9 A construction firm purchased 3 tractors from a certain company. At the end of the fifth year, let E_1, E_2, E_3 denote, respectively, the events that tractors no. 1, 2, and 3 are still in good operational condition.

(a) Define the following events at the end of the 5th year, in terms of E_1, E_2, and E_3, and their respective complements:

A = only tractor no. 1 is in good condition.

B = exactly one tractor is in good condition.

C = at least one tractor is in good condition.

(b) Past experience indicates that the chance of a given tractor manufactured by this company having a useful life longer than 5 years (i.e., in good condition at the end of the 5th year) is 60%. If one tractor needs to be replaced (not in good operational condition) at the end of the 5th year, the probability of replacement for one of the other two tractors is 60%; if two tractors need to be replaced, the probability of replacement of the remaining one is 80%.

Evaluate the probabilities of the events A, B, and C.

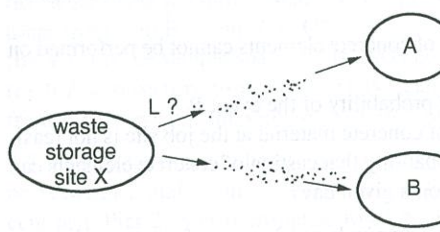
2.10 A contractor has two subcontractors for his excavation work. Experience shows that in 60% of the time, subcontractor A was available to do a job, whereas subcontractor B was available 80% of the time. Also, the contractor is able to get at least one of these two subcontractors 90% of the time.

(a) What is the probability that both subcontractors will be available to do the next job?

(b) If the contractor learned that subcontractor A is not available for the job, what is the probability that the other subcontractor will be available?

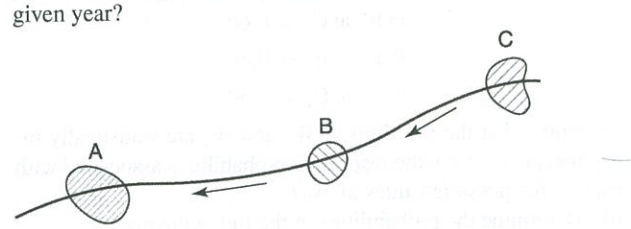
(c) Suppose E_A denotes the event that subcontractor A is available, and E_B denotes that subcontractor B is available.
(i) Are E_A and E_B statistically independent?
(ii) Are E_A and E_B mutually exhaustive?
(iii) Are E_A and E_B collectively exhaustive?

2.11 An underground site is being considered for the storage of hazardous waste. Within the next 100 years, there is a 1% chance that the hazardous material could leak outside of the storage containment. Two adjacent towns, A and B, rely on ground water for their water supply. The water to each town will be contaminated if there is a leakage in the waste storage and if there exists a continuous seam of sand between the storage containment and the given town. Observe that the presence of a continuous seam of sand would allow the contaminant to move freely and quickly access a region.



Suppose there is a 2% chance of a continuous seam of sand from the storage site to A, and the probability of a continuous seam of sand to B is slightly higher and equals 3%. However, if a continuous seam of sand exists between X and A, the probability of a continuous seam of sand between X and B is increased to 20%. Assume that the event of leakage from the storage is independent of the presence of seams of sand. Consider the period over the next 100 years.
(a) What is the probability that water in town A will be contaminated?
(b) What is the probability that at least one of the two towns' water will be contaminated?

2.12 Towns A, B, and C lie along a river, as shown in the figure below, which may be subject to overflow (flooding). The annual probabilities of flooding are 0.2, 0.3, and 0.1 for towns A, B, and C, respectively. The events of flooding in each of the towns A, B, and C are not statistically independent. If town C is flooded in a given year, the probability that town B will also be flooded that same year is increased to 0.6; and if both towns B and C are flooded in a given year, the probability that town A will also be flooded that year is increased to 0.8. However, if town C does not experience flooding in a given year, the probability that both towns A and B will also not suffer any flooding in that year is 0.9. In a given year, if all three towns are flooded, it is regarded as a *disaster year* for the year. Suppose the flooding events between any two years are statistically independent. Answer the following:
(a) What is the probability that a given year in the region is a disaster year?
(b) If town B is flooded in a given year, what is the probability that town C is also flooded?
(c) What is the probability that at least one town is flooded in a given year?



2.13 Successful completion of a construction project depends on the supply of materials and labor as well as the weather condition. Consider a given project that can be successfully completed if either one of the following conditions prevails:
(a) Good weather and at least labor or materials are adequately available.
(b) Bad weather but both labor and materials are adequately available.
Define:

G = Good weather
G′ = Bad weather
L = Adequate labor supply
M = Adequate materials supply
C = Successful completion

Suppose P(L) = 0.7; P(G) = 0.6 and L is independent of both M and G. If the weather is good, adequate supply of materials is guaranteed, whereas the probability of adequate supply of material is only 50% if bad weather prevails.
(a) Formulate the event of successful completion in terms of G, L, and M.
(b) Determine the probability of successful completion. (*Ans. 0.74*)
(c) If the project was successfully completed, what is the probability that labor supply had been inadequate? (*Ans. 0.243*)

2.14 The number of accidents at rail-highway grade-crossings reported for a province over the last 10 years are summarized and classified as follows:

		Type of Accident	
		(R) Run into Train	(S) Struck by Train
Time of Occurrence	Day (D)	30	60
	Night (N)	20	20

Suppose there are 1000 rail–highway grade-crossings in province XY.
(a) What is the probability that an accident will occur at a given crossing next year? (*Ans. 0.013*)

(b) If an accident is reported to have occurred in the daytime, what is the probability that it is a "struck by train" accident? (*Ans. 2/3*)
(c) Suppose that 50% of the "run into train" accidents are fatal and 80% of the "struck by train" accidents are fatal, what is the probability that the next accident will be fatal? (*Ans. 0.685*)
(d) Suppose D = event that the next accident occurs in the daytime
R = event that the next accident is a "run into train" accident
(i) Are D and R mutually exclusive? Justify.
(ii) Are D and R statistically independent? Justify. (*Ans. no*)

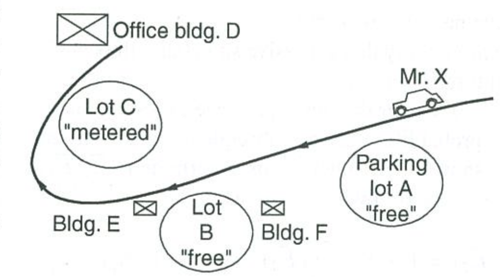
2.15 The promising alternative energy sources currently under development are fuel cell technology and large-scale solar energy power. The probabilities that these two sources will be successfully developed and commercially viable in the next 15 years are, respectively, 0.70 and 0.85. The successful development of these two energy sources are statistically independent. Determine the following:
(a) The probability that there will be energy supplied by these alternative sources in the next 15 years.
(b) The probability that only one of the two alternative energy sources will be commercially viable in the next 15 years.

2.16 An examination of the 10-year record of rainy days for a town reveals the following:
1. 30% of the days are rainy days.
2. There is a 50% chance that a rainy day will be followed by another rainy day.
3. There is a 20% chance that two consecutive rainy days will be followed by a third rainy day.
A house is scheduled for painting starting next Monday for a period of 3 days.
(a) Let E_1 = Monday is a rainy day
E_2 = Tuesday is a rainy day
E_3 = Wednesday is a rainy day

Express the events corresponding to the three probabilities indicated above; i.e., 1, 2, and 3, in terms of E_1, E_2, E_3.
(b) What is the probability that it will rain on both Monday and Tuesday?
(c) What is the probability that Wednesday will be the only dry day during the painting period?
(d) What is the probability that there will be at least one rainy day during the 3-day painting period?

2.17 Mr. X who works in the office building D is selected for observation in a study of the parking problem on a college campus. Each day, assume that Mr. X will check the parking lots *A*, *B*, and *C* in that sequence, as shown in the figure below, and will park his car as soon as he finds an empty space. Assume also that there are only these three parking lots available and no street parking is allowed, among which lots *A* and *B* are free—whereas lot *C* is metered.

Suppose that from prior statistical observations, the probabilities of getting a space on each weekday morning in lots *A*, *B*, and *C* are 0.20, 0.15, and 0.80, respectively. However, if lot *A* is full, the probability that Mr. X will find a space in lot *B* is only 0.05. Also, if both lots *A* and *B* are full, Mr. X will only have a probability of 40% of getting a parking space in lot *C*.
Determine the following:
(a) The probability that Mr. X will not find free parking on a weekday morning.
(b) The probability that Mr. X will be able to park his car on campus on a weekday morning.
(c) If Mr. X successfully parked his car on campus on a weekday morning, what is the probability that his parking is free?



A study of campus parking.

2.18 A building may fail by excessive settlement of the foundation or by collapse of the superstructure. Over the life of the building, the probability of excessive settlement of the foundation is estimated to be 0.10, whereas the probability of collapse of the superstructure is 0.05. Also, if there is excessive settlement of the foundation, the probability of superstructure collapse will be increased to 0.20.
(a) What is the probability that building failure will occur over its life?
(b) If building failure should occur during its life, what is the probability that both failure modes will occur?
(c) What is the probability that only one of the two failure modes will occur over the life of the building?

2.19 The automobile brake system consists of the following components: the master cylinder, the wheel cylinders, and the brake pads. Failure of any one or more of these components will constitute failure of the brake system. Within a period of 4 years or 50,000 miles without maintenance, the failure probabilities of the master cylinder, wheel cylinders, and brake pads are 0.02, 0.05, and 0.50, respectively. The probability that both the master cylinder and the wheel cylinders will fail within the same period or mileage is 0.01. The failure of the brake pads is statistically independent of the failures in the master and wheel cylinders.
(a) What is the probability that only the wheel cylinders will fail within 4 years or 50,000 miles?
(b) What is the probability that the brake system will fail within 4 years or 50,000 miles?
(c) When there is failure in the brake system, what is the probability that only one of the three components failed?