# Reference Sheet IV

Some of the important equations in this chapter are:

| | | |
|---|---|---|
| Conservation of mass | $\frac{\partial}{\partial t}\int_{cv}\rho\, d\forall + \int_{cs}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = 0$ | **(5.5)** |
| Mass flowrate | $\dot{m} = \rho Q = \rho A V$ | **(5.6)** |
| First law of thermodynamics (Conservation of energy) | $\frac{\partial}{\partial t}\int_{cv} e\rho\, d\forall + \int_{cs}\left(\breve{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz\right)\rho\mathbf{V}\cdot\hat{\mathbf{n}}\, dA = \dot{Q}_{\substack{\text{net}\\\text{in}}} + \dot{W}_{\substack{\text{shaft}\\\text{net in}}}$ | **(5.64)** |

Some of the important equations in this chapter are:

| | | |
|---|---|---|
| Ideal gas equation of state | $\rho = \frac{p}{RT}$ | **(11.1)** |
| Internal energy change | $\breve{u}_2 - \breve{u}_1 = c_v(T_2 - T_1)$ | **(11.5)** |
| Enthalpy | $\breve{h} = \breve{u} + \frac{p}{\rho}$ | **(11.6)** |
| Enthalpy change | $\breve{h}_2 - \breve{h}_1 = c_p(T_2 - T_1)$ | **(11.9)** |
| Specific heat difference | $c_p - c_v = R$ | **(11.12)** |
| Specific heat ratio | $k = \frac{c_p}{c_v}$ | **(11.13)** |
| Specific heat at constant pressure | $c_p = \frac{Rk}{k-1}$ | **(11.14)** |
| Specific heat at constant volume | $c_v = \frac{R}{k-1}$ | **(11.15)** |
| First $Tds$ equation | $T\, ds = d\breve{u} + p\, d\left(\frac{1}{\rho}\right)$ | **(11.16)** |
| Second $Tds$ equation | $T\, ds = d\breve{h} - \left(\frac{1}{\rho}\right)dp$ | **(11.18)** |
| Entropy change | $s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{\rho_1}{\rho_2}$ | **(11.21)** |
| Entropy change | $s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$ | **(11.22)** |
| Isentropic flow | $\frac{p}{\rho^k} = \text{constant}$ | **(11.25)** |
| Speed of sound | $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$ | **(11.34)** |
| Speed of sound in gas | $c = \sqrt{RTk}$ | **(11.36)** |
| Speed of sound in liquid | $c = \sqrt{\frac{E_v}{\rho}}$ | **(11.38)** |
| Mach cone angle | $\sin\alpha = \frac{c}{V} = \frac{1}{\text{Ma}}$ | **(11.39)** |
| Mach number | $\text{Ma} = \frac{V}{c}$ | **(11.46)** |

# Isentropic Flow Relations (Ideal Gas with $k = 1.4$)

Isentropic flow

$$\frac{T}{T_0} = \frac{1}{1 + [(k-1)/2]\text{Ma}^2} \tag{11.56}$$

Isentropic flow

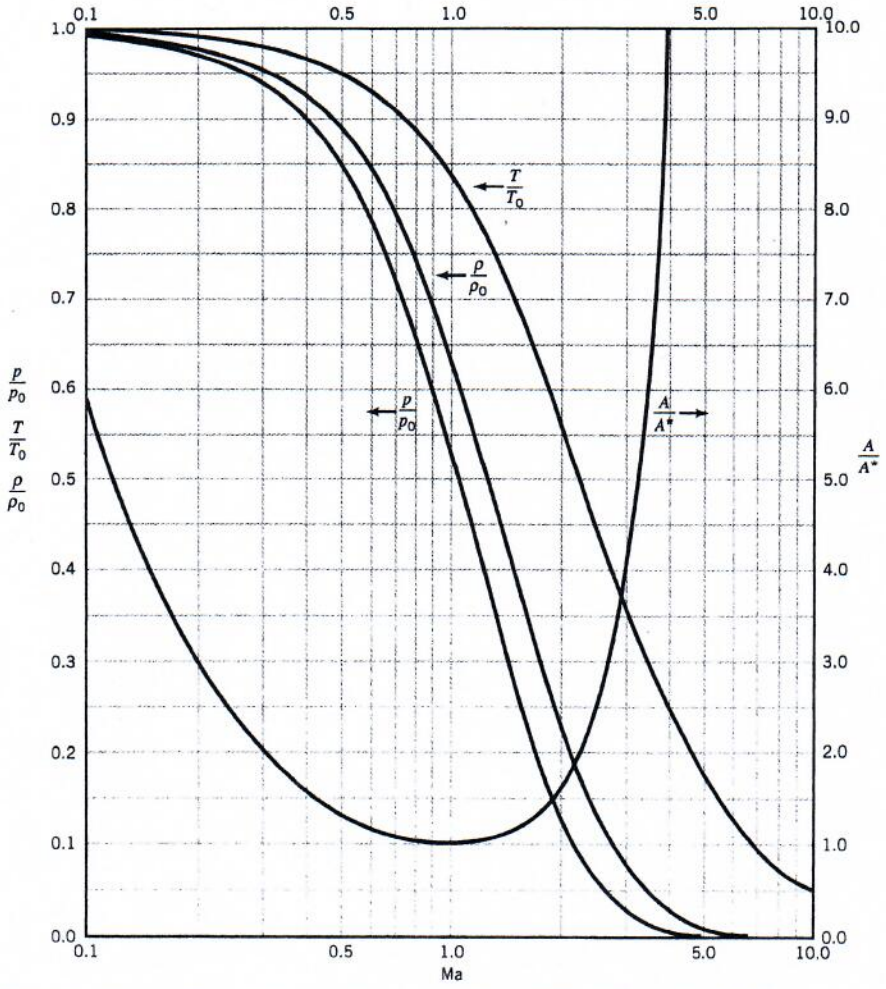
$$\frac{p}{p_0} = \left\{ \frac{1}{1 + [(k-1)/2]\text{Ma}^2} \right\}^{k/(k-1)} \tag{11.59}$$

Isentropic flow

$$\frac{\rho}{\rho_0} = \left\{ \frac{1}{1 + [(k-1)/2]\text{Ma}^2} \right\}^{1/(k-1)} \tag{11.60}$$

Isentropic flow

$$\frac{A}{A^*} = \frac{1}{\text{Ma}} \left\{ \frac{1 + [(k-1)/2]\text{Ma}^2}{1 + [(k-1)/2]} \right\}^{(k+1)/[2(k-1)]} \tag{11.71}$$




■ **Figure D.1** Isentropic flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)

## Table D.1

**Isentropic Flow Functions for an Ideal Gas with $k = 1.4$**

| Ma | $T/T_0$ | $p/p_0$ | $\rho/\rho_0$ | $A/A^*$ | Ma | $T/T_0$ | $p/p_0$ | $\rho/\rho_0$ | $A/A^*$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 1.0000 | 1.0000 | 1.0000 | $\infty$ | 2.0500 | 0.5433 | 0.1182 | 0.2176 | 1.7600 |
| 0.0500 | 0.9995 | 0.9983 | 0.9988 | 11.5914 | 2.1000 | 0.5313 | 0.1094 | 0.2058 | 1.8369 |
| 0.1000 | 0.9980 | 0.9930 | 0.9950 | 5.8218 | 2.1500 | 0.5196 | 0.1011 | 0.1946 | 1.9185 |
| 0.1500 | 0.9955 | 0.9844 | 0.9888 | 3.9103 | 2.2000 | 0.5081 | 0.0935 | 0.1841 | 2.0050 |
| 0.2000 | 0.9921 | 0.9725 | 0.9803 | 2.9635 | 2.2500 | 0.4969 | 0.0865 | 0.1740 | 2.0964 |
| 0.2500 | 0.9877 | 0.9575 | 0.9694 | 2.4027 | 2.3000 | 0.4859 | 0.0800 | 0.1646 | 2.1931 |
| 0.3000 | 0.9823 | 0.9395 | 0.9564 | 2.0351 | 2.3500 | 0.4752 | 0.0740 | 0.1556 | 2.2953 |
| 0.3500 | 0.9761 | 0.9188 | 0.9413 | 1.7780 | 2.4000 | 0.4647 | 0.0684 | 0.1472 | 2.4031 |
| 0.4000 | 0.9690 | 0.8956 | 0.9243 | 1.5901 | 2.4500 | 0.4544 | 0.0633 | 0.1392 | 2.5168 |
| 0.4500 | 0.9611 | 0.8703 | 0.9055 | 1.4487 | 2.5000 | 0.4444 | 0.0585 | 0.1317 | 2.6367 |
| 0.5000 | 0.9524 | 0.8430 | 0.8852 | 1.3398 | 2.5500 | 0.4347 | 0.0542 | 0.1246 | 2.7630 |
| 0.5500 | 0.9430 | 0.8142 | 0.8634 | 1.2549 | 2.6000 | 0.4252 | 0.0501 | 0.1179 | 2.8960 |
| 0.6000 | 0.9328 | 0.7840 | 0.8405 | 1.1882 | 2.6500 | 0.4159 | 0.0464 | 0.1115 | 3.0359 |
| 0.6500 | 0.9221 | 0.7528 | 0.8164 | 1.1356 | 2.7000 | 0.4068 | 0.0430 | 0.1056 | 3.1830 |
| 0.7000 | 0.9107 | 0.7209 | 0.7916 | 1.0944 | 2.7500 | 0.3980 | 0.0398 | 0.0999 | 3.3377 |
| 0.7500 | 0.8989 | 0.6886 | 0.7660 | 1.0624 | 2.8000 | 0.3894 | 0.0368 | 0.0946 | 3.5001 |
| 0.8000 | 0.8865 | 0.6560 | 0.7400 | 1.0382 | 2.8500 | 0.3810 | 0.0341 | 0.0896 | 3.6707 |
| 0.8500 | 0.8737 | 0.6235 | 0.7136 | 1.0207 | 2.9000 | 0.3729 | 0.0317 | 0.0849 | 3.8498 |
| 0.9000 | 0.8606 | 0.5913 | 0.6870 | 1.0089 | 2.9500 | 0.3649 | 0.0293 | 0.0804 | 4.0376 |
| 0.9500 | 0.8471 | 0.5595 | 0.6604 | 1.0021 | 3.0000 | 0.3571 | 0.0272 | 0.0762 | 4.2346 |
| 1.0000 | 0.8333 | 0.5283 | 0.6339 | 1.0000 | 3.1000 | 0.3422 | 0.0234 | 0.0685 | 4.6573 |
| 1.0500 | 0.8193 | 0.4979 | 0.6077 | 1.0020 | 3.2000 | 0.3281 | 0.0202 | 0.0617 | 5.1210 |
| 1.1000 | 0.8052 | 0.4684 | 0.5817 | 1.0079 | 3.3000 | 0.3147 | 0.0175 | 0.0555 | 5.6286 |
| 1.1500 | 0.7908 | 0.4398 | 0.5562 | 1.0175 | 3.4000 | 0.3019 | 0.0151 | 0.0501 | 6.1837 |
| 1.2000 | 0.7764 | 0.4124 | 0.5311 | 1.0304 | 3.5000 | 0.2899 | 0.0131 | 0.0452 | 6.7896 |
| 1.2500 | 0.7619 | 0.3861 | 0.5067 | 1.0468 | 3.6000 | 0.2784 | 0.0114 | 0.0409 | 7.4501 |
| 1.3000 | 0.7474 | 0.3609 | 0.4829 | 1.0663 | 3.7000 | 0.2675 | 0.0099 | 0.0370 | 8.1691 |
| 1.3500 | 0.7329 | 0.3370 | 0.4598 | 1.0890 | 3.8000 | 0.2572 | 0.0086 | 0.0335 | 8.9506 |
| 1.4000 | 0.7184 | 0.3142 | 0.4374 | 1.1149 | 3.9000 | 0.2474 | 0.0075 | 0.0304 | 9.7990 |
| 1.4500 | 0.7040 | 0.2927 | 0.4158 | 1.1440 | 4.0000 | 0.2381 | 0.0066 | 0.0277 | 10.7188 |
| 1.5000 | 0.6897 | 0.2724 | 0.3950 | 1.1762 | 4.5000 | 0.1980 | 0.0035 | 0.0174 | 16.5622 |
| 1.5500 | 0.6754 | 0.2533 | 0.3750 | 1.2116 | 5.0000 | 0.1667 | 0.0019 | 0.0113 | 25.0000 |
| 1.6000 | 0.6614 | 0.2353 | 0.3557 | 1.2502 | 5.5000 | 0.1418 | 0.0011 | 0.0076 | 36.8690 |
| 1.6500 | 0.6475 | 0.2184 | 0.3373 | 1.2922 | 6.0000 | 0.1220 | 0.0006 | 0.0052 | 53.1798 |
| 1.7000 | 0.6337 | 0.2026 | 0.3197 | 1.3376 | 6.5000 | 0.1058 | 0.0004 | 0.0036 | 75.1343 |
| 1.7500 | 0.6202 | 0.1878 | 0.3029 | 1.3865 | 7.0000 | 0.0926 | 0.0002 | 0.0026 | 104.1429 |
| 1.8000 | 0.6068 | 0.1740 | 0.2868 | 1.4390 | 7.5000 | 0.0816 | 0.0002 | 0.0019 | 141.8415 |
| 1.8500 | 0.5936 | 0.1612 | 0.2715 | 1.4952 | 8.0000 | 0.0725 | 0.0001 | 0.0014 | 190.1094 |
| 1.9000 | 0.5807 | 0.1492 | 0.2570 | 1.5553 | 8.5000 | 0.0647 | 0.0001 | 0.0011 | 251.0862 |
| 1.9500 | 0.5680 | 0.1381 | 0.2432 | 1.6193 | 9.0000 | 0.0581 | 0.0000 | 0.0008 | 327.1893 |
| 2.0000 | 0.5556 | 0.1278 | 0.2300 | 1.6875 | 10.0000 | 0.0476 | 0.0000 | 0.0005 | 535.9375 |

# Normal Shock Relations (Ideal Gas with $k = 1.4$)

Normal shock

$$\mathrm{Ma}_y^2 = \frac{\mathrm{Ma}_x^2 + [2/(k-1)]}{[2k/(k-1)]\mathrm{Ma}_x^2 - 1} \tag{11.149}$$

Normal shock

$$\frac{p_y}{p_x} = \frac{2k}{k+1}\mathrm{Ma}_x^2 - \frac{k-1}{k+1} \tag{11.150}$$

Normal shock

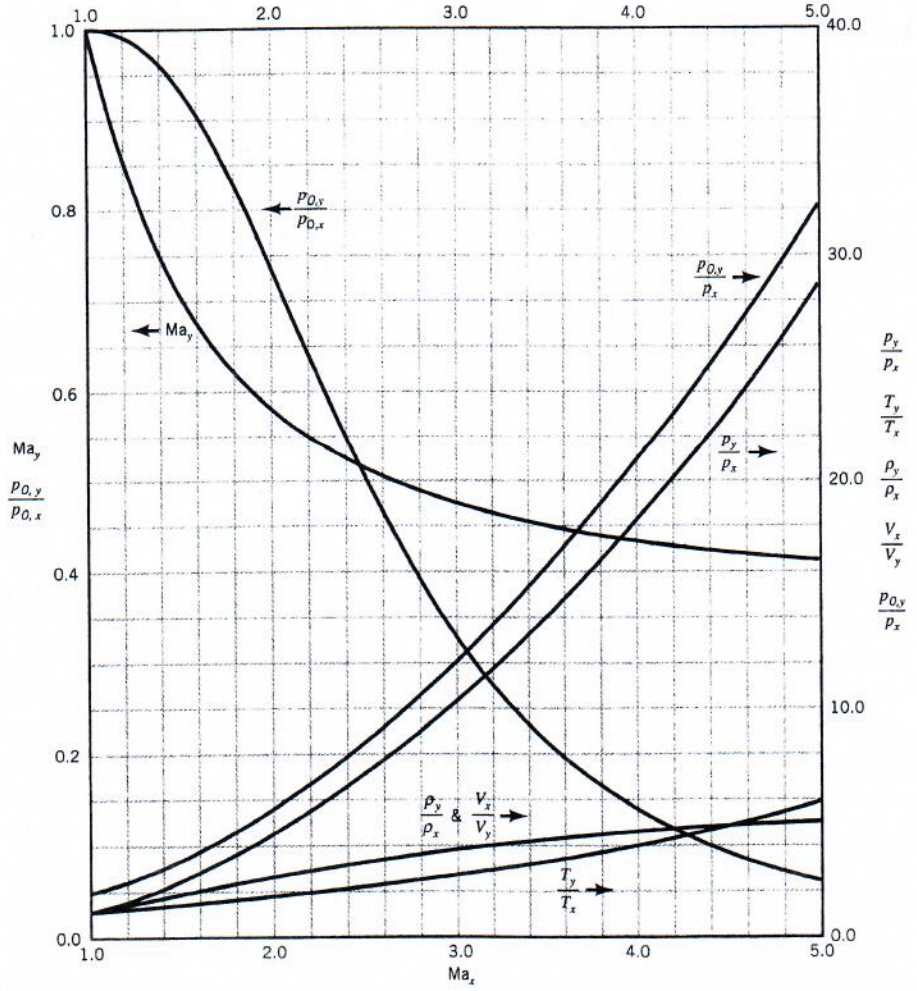
$$\frac{T_y}{T_x} = \frac{\{1 + [(k-1)/2]\mathrm{Ma}_x^2\}\{[2k/(k-1)]\mathrm{Ma}_x^2 - 1\}}{\{(k+1)^2/[2(k-1)]\}\mathrm{Ma}_x^2} \tag{11.151}$$

Normal shock

$$\frac{\rho_y}{\rho_x} = \frac{V_x}{V_y} = \frac{(k+1)\mathrm{Ma}_x^2}{(k-1)\mathrm{Ma}_x^2 + 2} \tag{11.154}$$

Normal shock

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left(\frac{k+1}{2}\mathrm{Ma}_x^2\right)^{k/(k-1)}\left(1 + \frac{k-1}{2}\mathrm{Ma}_x^2\right)^{k/(1-k)}}{\left(\frac{2k}{k+1}\mathrm{Ma}_x^2 - \frac{k-1}{k+1}\right)^{1/(k-1)}} \tag{11.156}$$




■ **Figure D.4** Normal shock flow of an ideal gas with $k = 1.4$. (Graph provided by Dr. Bruce A. Reichert.)

## Table D.S

**Normal Shock Functions for an Ideal Gas with $k = 1.4$**

| $Ma_x$ | $Ma_y$ | $p_y/p_x$ | $T_y/T_x$ | $\rho_y/\rho_x = V_x/V_y$ | $p_{0y}/p_{0x}$ | $Ma_x$ | $Ma_y$ | $p_y/p_x$ | $T_y/T_x$ | $\rho_y/\rho_x = V_x/V_y$ | $p_{0y}/p_{0x}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.00 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 3.05 | 0.4723 | 10.6863 | 2.7383 | 3.9025 | 0.3145 |
| 1.05 | 0.9531 | 1.1196 | 1.0328 | 1.0840 | 0.9999 | 3.10 | 0.4695 | 11.0450 | 2.7986 | 3.9466 | 0.3012 |
| 1.10 | 0.9118 | 1.2450 | 1.0649 | 1.1691 | 0.9989 | 3.15 | 0.4669 | 11.4096 | 2.8598 | 3.9896 | 0.2885 |
| 1.15 | 0.8750 | 1.3763 | 1.0966 | 1.2550 | 0.9967 | 3.20 | 0.4643 | 11.7800 | 2.9220 | 4.0315 | 0.2762 |
| 1.20 | 0.8422 | 1.5133 | 1.1280 | 1.3416 | 0.9928 | 3.25 | 0.4619 | 12.1563 | 2.9851 | 4.0723 | 0.2645 |
| 1.25 | 0.8126 | 1.6563 | 1.1594 | 1.4286 | 0.9871 | 3.30 | 0.4596 | 12.5383 | 3.0492 | 4.1120 | 0.2533 |
| 1.30 | 0.7860 | 1.8050 | 1.1909 | 1.5157 | 0.9794 | 3.35 | 0.4573 | 12.9263 | 3.1142 | 4.1507 | 0.2425 |
| 1.35 | 0.7618 | 1.9596 | 1.2226 | 1.6028 | 0.9697 | 3.40 | 0.4552 | 13.3200 | 3.1802 | 4.1884 | 0.2322 |
| 1.40 | 0.7397 | 2.1200 | 1.2547 | 1.6897 | 0.9582 | 3.45 | 0.4531 | 13.7196 | 3.2472 | 4.2251 | 0.2224 |
| 1.45 | 0.7196 | 2.2863 | 1.2872 | 1.7761 | 0.9448 | 3.50 | 0.4512 | 14.1250 | 3.3151 | 4.2609 | 0.2129 |
| 1.50 | 0.7011 | 2.4583 | 1.3202 | 1.8621 | 0.9298 | 3.55 | 0.4492 | 14.5363 | 3.3839 | 4.2957 | 0.2039 |
| 1.55 | 0.6841 | 2.6363 | 1.3538 | 1.9473 | 0.9132 | 3.60 | 0.4474 | 14.9533 | 3.4537 | 4.3296 | 0.1953 |
| 1.60 | 0.6684 | 2.8200 | 1.3880 | 2.0317 | 0.8952 | 3.65 | 0.4456 | 15.3763 | 3.5245 | 4.3627 | 0.1871 |
| 1.65 | 0.6540 | 3.0096 | 1.4228 | 2.1152 | 0.8760 | 3.70 | 0.4439 | 15.8050 | 3.5962 | 4.3949 | 0.1792 |
| 1.70 | 0.6405 | 3.2050 | 1.4583 | 2.1977 | 0.8557 | 3.75 | 0.4423 | 16.2396 | 3.6689 | 4.4262 | 0.1717 |
| 1.75 | 0.6281 | 3.4063 | 1.4946 | 2.2791 | 0.8346 | 3.80 | 0.4407 | 16.6800 | 3.7426 | 4.4568 | 0.1645 |
| 1.80 | 0.6165 | 3.6133 | 1.5316 | 2.3592 | 0.8127 | 3.85 | 0.4392 | 17.1263 | 3.8172 | 4.4866 | 0.1576 |
| 1.85 | 0.6057 | 3.8263 | 1.5693 | 2.4381 | 0.7902 | 3.90 | 0.4377 | 17.5783 | 3.8928 | 4.5156 | 0.1510 |
| 1.90 | 0.5956 | 4.0450 | 1.6079 | 2.5157 | 0.7674 | 3.95 | 0.4363 | 18.0363 | 3.9694 | 4.5439 | 0.1448 |
| 1.95 | 0.5862 | 4.2696 | 1.6473 | 2.5919 | 0.7442 | 4.00 | 0.4350 | 18.5000 | 4.0469 | 4.5714 | 0.1388 |
| 2.00 | 0.5774 | 4.5000 | 1.6875 | 2.6667 | 0.7209 | 4.10 | 0.4324 | 19.4450 | 4.2048 | 4.6245 | 0.1276 |
| 2.05 | 0.5691 | 4.7363 | 1.7285 | 2.7400 | 0.6975 | 4.20 | 0.4299 | 20.4133 | 4.3666 | 4.6749 | 0.1173 |
| 2.10 | 0.5613 | 4.9783 | 1.7705 | 2.8119 | 0.6742 | 4.30 | 0.4277 | 21.4050 | 4.5322 | 4.7229 | 0.1080 |
| 2.15 | 0.5540 | 5.2263 | 1.8132 | 2.8823 | 0.6511 | 4.40 | 0.4255 | 22.4200 | 4.7017 | 4.7685 | 0.0995 |
| 2.20 | 0.5471 | 5.4800 | 1.8569 | 2.9512 | 0.6281 | 4.50 | 0.4236 | 23.4583 | 4.8751 | 4.8119 | 0.0917 |
| 2.25 | 0.5406 | 5.7396 | 1.9014 | 3.0186 | 0.6055 | 4.60 | 0.4217 | 24.5200 | 5.0523 | 4.8532 | 0.0846 |
| 2.30 | 0.5344 | 6.0050 | 1.9468 | 3.0845 | 0.5833 | 4.70 | 0.4199 | 25.6050 | 5.2334 | 4.8926 | 0.0781 |
| 2.35 | 0.5286 | 6.2763 | 1.9931 | 3.1490 | 0.5615 | 4.80 | 0.4183 | 26.7133 | 5.4184 | 4.9301 | 0.0721 |
| 2.40 | 0.5231 | 6.5533 | 2.0403 | 3.2119 | 0.5401 | 4.90 | 0.4167 | 27.8450 | 5.6073 | 4.9659 | 0.0667 |
| 2.45 | 0.5179 | 6.8363 | 2.0885 | 3.2733 | 0.5193 | 5.00 | 0.4152 | 29.0000 | 5.8000 | 5.0000 | 0.0617 |
| 2.50 | 0.5130 | 7.1250 | 2.1375 | 3.3333 | 0.4990 | 5.50 | 0.4090 | 35.1250 | 6.8218 | 5.1489 | 0.0424 |
| 2.55 | 0.5083 | 7.4196 | 2.1875 | 3.3919 | 0.4793 | 6.00 | 0.4042 | 41.8333 | 7.9406 | 5.2683 | 0.0297 |
| 2.60 | 0.5039 | 7.7200 | 2.2383 | 3.4490 | 0.4601 | 6.50 | 0.4004 | 49.1250 | 9.1564 | 5.3651 | 0.0211 |
| 2.65 | 0.4996 | 8.0262 | 2.2902 | 3.5047 | 0.4416 | 7.00 | 0.3974 | 57.0000 | 10.4694 | 5.4444 | 0.0154 |
| 2.70 | 0.4956 | 8.3383 | 2.3429 | 3.5590 | 0.4236 | 7.50 | 0.3949 | 65.4583 | 11.8795 | 5.5102 | 0.0113 |
| 2.75 | 0.4918 | 8.6562 | 2.3966 | 3.6119 | 0.4062 | 8.00 | 0.3929 | 74.5000 | 13.3867 | 5.5652 | 0.0085 |
| 2.80 | 0.4882 | 8.9800 | 2.4512 | 3.6636 | 0.3895 | 8.50 | 0.3912 | 84.1250 | 14.9911 | 5.6117 | 0.0064 |
| 2.85 | 0.4847 | 9.3096 | 2.5067 | 3.7139 | 0.3733 | 9.00 | 0.3898 | 94.3333 | 16.6927 | 5.6512 | 0.0050 |
| 2.90 | 0.4814 | 9.6450 | 2.5632 | 3.7629 | 0.3577 | 9.50 | 0.3886 | 105.1250 | 18.4915 | 5.6850 | 0.0039 |
| 2.95 | 0.4782 | 9.9862 | 2.6206 | 3.8106 | 0.3428 | 10.00 | 0.3876 | 116.5000 | 20.3875 | 5.7143 | 0.0030 |
| 3.00 | 0.4752 | 10.3333 | 2.6790 | 3.8571 | 0.3283 | $\infty$ | 0.3780 | $\infty$ | $\infty$ | 6.0000 | 0.0000 |